Make Breakthrough Improvements

with Design of Experiments

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Executive summary

In many rubber and plastics processes, powerful interactions affect final performance. You will not discover interactions when you change only one factor at a time. Proper design of experiments (DOE) will reveal interactions that can help you achieve breakthrough improvements in process efficiency and product quality. The big gains come from a very simple form of DOE called two-level factorial design. This approach to experimentation has proven to be especially helpful for control of part shrinkage, but it can be applied to any measurable response. In this article you will be given the primary details. We will do this from an engineering perspective, with an emphasis on the practical aspects.

Two-level factorials for maximum efficiency

Two-level factorial design is a statistically-based method that involves simultaneous adjustment of experimental factors at only two levels: high and low. The two-level design approach offers a parallel testing scheme that's much more efficient than one-factor-at-a-time. By restricting the tests to only two levels, you minimize the number of experiments. The contrast between levels gives you the necessary driving force for product improvement. Aside from the issue of efficiency, two-level factorial designs can uncover critical interactions that wouldn't be revealed any other way.

Two-level factorial designs can be constructed with the aid of a textbook[1], or better yet, with a statistical software package, most of which offer design of experiments capabilities[2]. You will find designs available with as little as one more run than the number of factors you want to test. For example you could test 7 factors in 8 runs, or 15 factors in 16 runs. However, these "saturated" designs provide very poor resolution: main effects will be confused with two-factor interactions. We advise that you avoid running such low resolution designs.

DOE reveals two-factor interactions affecting part shrinkage

An injection molder seeking data on shrinkage set aside one production line for a study of 7 factors (Table 1). All possible combinations of these factors would require 128 experiments, but only 32 of these were actually run - a 1/4th fraction. This DOE is symbolized mathematically as 2^{7-2}. From the 32 runs you can get information on all the main effects and nearly all two-factor interactions. Table 2 shows the experimental design matrix in terms of coded factor levels: –1 for low and +1 for high. The experiment was divided into 4 blocks of 8 runs in a way that preserved the information on main effects and interactions. The experimenters then ran the DOE on 4 parallel lines, thus greatly reducing the time needed to generate the data, as well as providing information on machine-to-machine variation.

Note the balanced array of plus (high) and minus (low) levels in the test matrix. Each column contains 16 pluses and 16 minuses. The matrix offers a very important statistical property called “orthogonality” which means that factors are not correlated. If you just collected happenstance data from production records, it is highly unlikely you would get an array of factors like this. You would probably find that factors such as temperature and pressure go up and down together. As factors become more and more correlated, the error in estimation of their effects becomes larger and larger. That’s not good.

Orthogonal test matrices make effect estimation neat and easy. For example, the effect of factor A is calculated by simply averaging the responses at the plus level and subtracting the average at the minus levels.

Using statistical principles to pick significant factors

You now know how to calculate effects. It seems obvious that you should pick the largest ones and run with those - right? Wrong! How do you know where to make the cut-off? What if none of the effects are real, and you’ve just measured results due to random error? Somehow the vital few significant factors must be screened out of the trivial many that occur due to chance. You can do this easily with a graph called a “normal plot”. Textbooks provide details on how to construct these graphs, but you will find it much easier to let DOE software do it for you. Typically you will see a group of near-zero effects that form a line. Then after a noticeable gap you may find effects that are much smaller or much larger than the others. Anything significant will fall off to the left or right of the line. Figure 1 shows the normal plot of effects for the molding case. Significant effects are labeled. The near-zero effects fall on a straight line - exhibiting normal scatter. These insignificant effects can be used to estimate experimental error.

If you want to be conservative, consider replicating the design to get estimates of “pure” error. Be sure to randomize the run order of your entire design, including replicates. Otherwise you leave yourself open to “lurking factors”, such as gradual change in ambient temperature or machine wear, that could confound your factor estimates.

Given a valid estimate of experimental error, regardless of the source, standard statistical analyses can then be performed to validate the overall outcome and individual effects. Textbooks provide hand-calculation schemes for doing statistical analysis of two-level factorials, but it’s much easier to let a statistical software program to do this work for you.

The statistical analysis may reveal individual outliers. But be careful, don’t delete points unless you can assign a special cause, such as a temporary equipment breakdown or the like. Quite often an outlier turns out to be simply an error in data entry. Digits can get transposed very easily.

Interpreting the results

Now you are ready to make your report. Start by making a plot of any significant main effects that are not part of a significant interaction. In this case there are none that stand alone. Next, produce the interaction plots, for example BF (Figure 2). Notice that the lines on this plot are not parallel. In other words, the effect of one factor depends on the level of the other. One-factor-at-a-time experiments would find the main effects of B (holding pressure) and F (cycle time) to be negligible. The 2-level factorial revealed a powerful interaction between these factors, making it obvious that it is inappropriate to study these factors individually. The best combination for minimum shrinkage with maximum throughput is B– (low holding pressure) and F– (low cycle time). These results remained true regardless of the machine lines, which exhibited only minor variation from block to block.

Finding robust operating conditions

The DOE on the molding operation revealed another significant interaction: CD (Figure 3). Notice that C (booster pressure) has an effect only when D (moisture) is high. Clearly if you want to make shrinkage robust to booster pressure, it's best to keep moisture low (D–).

Before you make a final recommendation on the new factor levels, it would be wise to perform confirmation runs. However, be prepared for some variation when you do confirmation tests. Your software should provide a confidence interval on the expected values. Use this data to "manage expectations".

What's in it for you

The case study on molding illustrates how two-level factorials can be applied to a process with many variables. The design of experiments uncovered large interactions which led to a breakthrough improvement. The experimenters found that many of the factors had no effect, so they could be set at their most economical level. Thus, significant savings can be achieved even from statistically insignificant factors. Another interesting aspect of this case is that one of the factors, moisture, could be set to a level that makes the process robust to the variation in another factor, thus making control of shrinkage much easier.

The 2-level factorial approach is very effective as a screening tool. After identifying the vital few factors you can then move on to more in-depth study via response surface methods (RSM).[3] Then you can reach the absolute peak of performance. Figure 4 shows the response surface for the BF interaction with D (moisture) set at its low level.[4] Further study of these factors via RSM might reveal a more complex relation.

If you equip yourself with the tools of statistical DOE, the competitive position of your company will be advanced, and your technical reputation will be enhanced.

Literature Cited

(1) Montgomery, D. C., *Design and Analysis of Experiments*, 4th ed., John Wiley & Sons, Inc, New York, 1997.

(2) Helseth, T.J., et al, *Design-Ease*, Windows or Macintosh, Stat-Ease, Inc, Minneapolis ($395).
(3) Montgomery, D. C., Myers, R. H., *Response Surface Methods*, John Wiley & Sons, Inc, New York, 1995.
(4) Helseth, T. J., et al, *Design-Expert*, Windows, Stat-Ease, Inc, Minneapolis ($995).

Table 1. Factors for Molding Case

	Factor	Units	Low (–)	High (+)
A.	Mold temperature	degrees F	130	180
B.	Holding Pressure	psig	1200	1500
C.	Booster Pressure	psig	1500	1800
D.	Moisture	percent	0.05	0.15
E.	Screw speed	inches/sec	1.5	4.0
F.	Cycle time	seconds	25	30
G.	Gate size	thousands	30	50

Table 2. Experimental Design Matrix and Data for Molding Case

Std	Run	Line	A	B	C	D	E	F	G	Shrinkage (%)
1	11	2	–1	–1	–1	–1	–1	+1	+1	20.3
2	21	3	+1	–1	–1	–1	–1	–1	–1	17.9
3	4	1	–1	+1	–1	–1	–1	–1	–1	22.1
4	28	4	+1	+1	–1	–1	–1	+1	+1	17.9
5	26	4	–1	–1	+1	–1	–1	–1	+1	17.5
6	2	1	+1	–1	+1	–1	–1	+1	–1	21.5
7	17	3	–1	+1	+1	–1	–1	+1	–1	18.3
8	12	2	+1	+1	+1	–1	–1	–1	+1	22.3
9	14	2	–1	–1	–1	+1	–1	–1	–1	15.0
10	23	3	+1	–1	–1	+1	–1	+1	+1	18.1
11	5	1	–1	+1	–1	+1	–1	+1	+1	15.0
12	31	4	+1	+1	–1	+1	–1	–1	–1	16.9
13	27	4	–1	–1	+1	+1	–1	+1	–1	27.6
14	3	1	+1	–1	+1	+1	–1	–1	+1	24.2
15	18	3	–1	+1	+1	+1	–1	–1	+1	28.7
16	16	2	+1	+1	+1	+1	–1	+1	–1	22.4
17	32	4	–1	–1	–1	–1	+1	+1	–1	20.2
18	1	1	+1	–1	–1	–1	+1	–1	+1	17.6
19	20	3	–1	+1	–1	–1	+1	–1	+1	21.5
20	9	2	+1	+1	–1	–1	+1	+1	–1	16.3
21	13	2	–1	–1	+1	–1	+1	–1	–1	17.7
22	24	3	+1	–1	+1	–1	+1	+1	+1	20.9
23	6	1	–1	+1	+1	–1	+1	+1	+1	17.8
24	29	4	+1	+1	+1	–1	+1	–1	–1	21.9
25	30	4	–1	–1	–1	+1	+1	–1	+1	14.8
26	8	1	+1	–1	–1	+1	+1	+1	–1	17.3
27	22	3	–1	+1	–1	+1	+1	+1	–1	16.1
28	15	2	+1	+1	–1	+1	+1	–1	+1	16.5
29	10	2	–1	–1	+1	+1	+1	+1	+1	28.2

30	19	3	+1	–1	+1	+1	+1	–1	–1	23.1
31	7	1	–1	+1	+1	+1	+1	–1	–1	27.6
32	25	4	+1	+1	+1	+1	+1	+1	+1	22.4

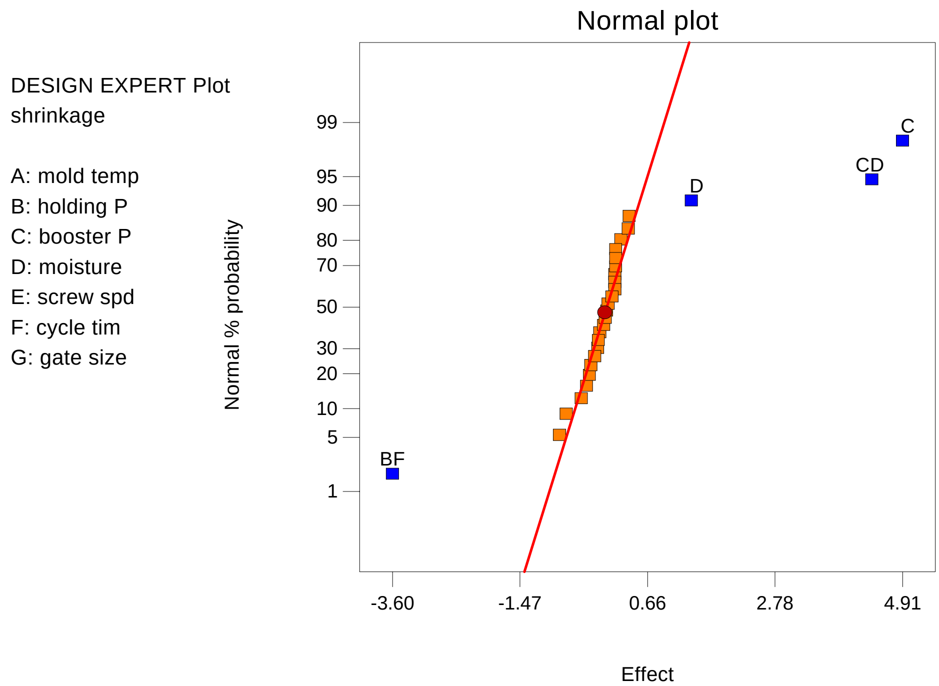

Figure 1. Normal Plot of Effects

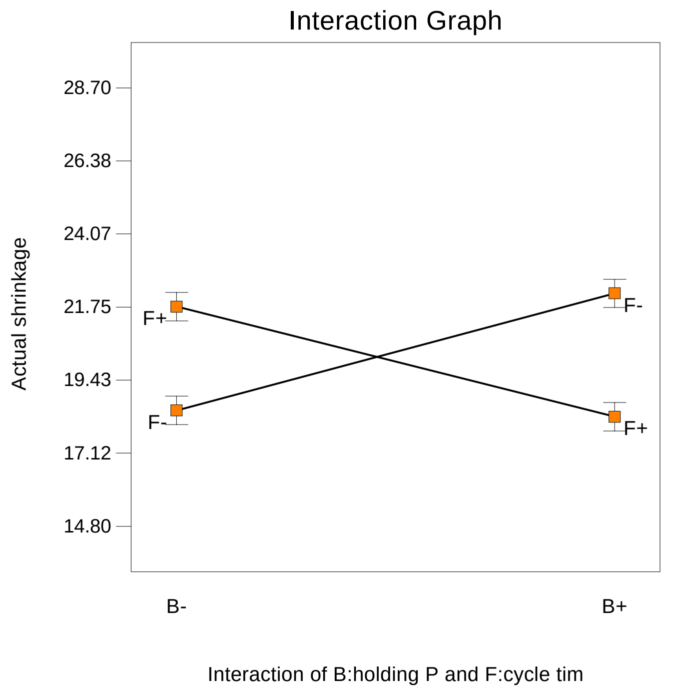

Figure 2. Interpretation Plot for Interaction BF

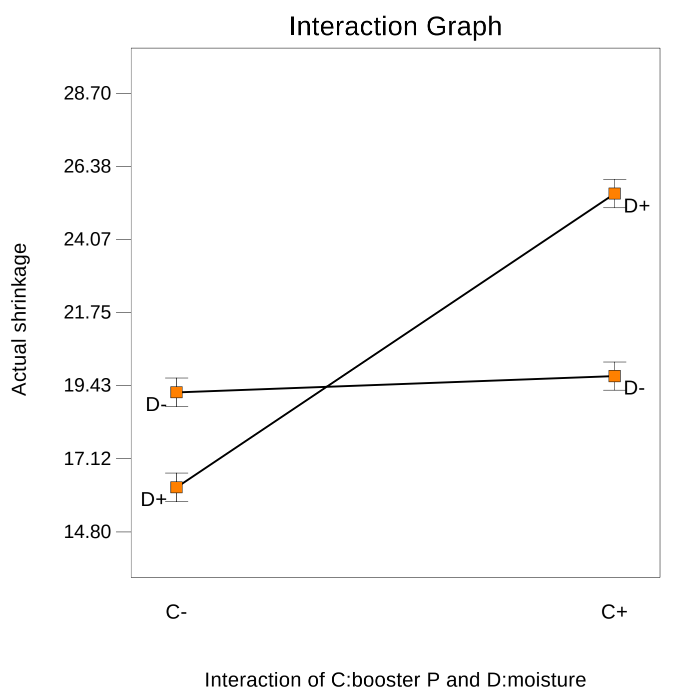

Figure 3. Interpretation Plot for Interaction CD

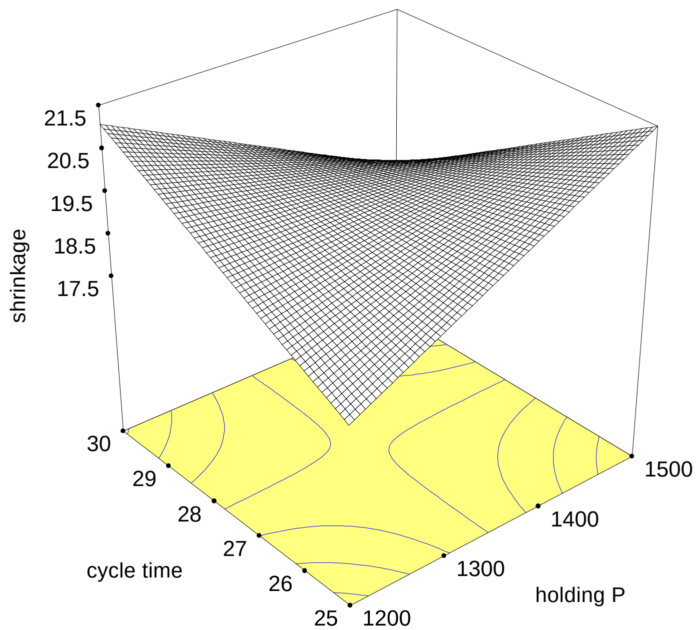

Figure 4. Response Surface Graph for BF (with D set low)