

Section 3 – Two-Level Factorial Tutorials

Full Factorial

Introduction

This tutorial demonstrates the use of Design-Expert® software for two-level factorial designs. These designs will help you screen many factors to discover the vital few, and perhaps how they interact. If you haven't done the one-factor tutorial that precedes this, please do so now, because it will be assumed that you already have some familiarity with the software.

The data you will now analyze comes from Douglas Montgomery's textbook, *Design and Analysis of Experiments*, published by John Wiley and Sons, New York. A chemical product is produced in a pressure vessel. A full-factorial experiment is carried out in the pilot plant to study four factors thought to influence the filtration rate of the product. The following table shows actual high and low levels for each of the factors.

Factor	Units	Low Level (-)	High Level (+)
A. Temperature	deg C	24	35
B. Pressure	Psig	10	15
C. Concentration	Percent	2	4
D. Stir Rate	Rpm	15	30

Factors and Levels for Full Factorial Design Example

At each combination of these machine settings, the experimenters recorded the filtration rate. The goal is to maximize the filtration rate and also try to find conditions that would allow a reduction in the concentration of formaldehyde, factor C. This case study exercises many of the two-level design features offered by Design-Expert. It should get you well down the road to being a power user. Let's get going!

Design the Experiment

Start the program by finding and double-clicking the Design-Expert icon.

Click on **F**ile in the menu bar and select **N**ew Design. You will see four tabs on the left of your screen: Stay with the **F**actorial choice, which comes up by default. You'll be using the default selection – **2 Level Factorial**.

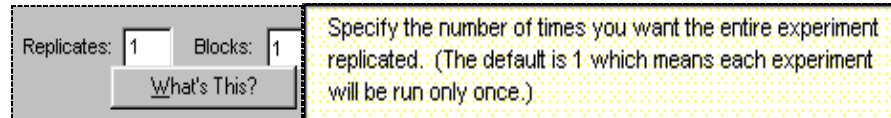
The default design builder offers full and fractional two-level factorials for 2 to 15 factors in 4, 8, 16, 32, 64, 128, or 256 experiments. The choices come up in color on your screen. The white squares symbolize full factorials (all combinations). The other choices are colored like a stoplight: green for go, yellow for proceed with caution and red for stop, which represent varying degrees of resolution: $\geq V$, IV, and III, respectively. For complete details on fractional factorials, and the concept of resolution, refer to the reference by Montgomery.

As shown below, start the design building process by clicking on the **Full** square under column **4** for number of factors, in the experiments row labeled **16**.

		Number of Factors												
		2	3	4	5	6	7	8	9	10	11	12	13	14
Experiments	4	Full	1/2 Fract.											
	8		Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.							
	16			Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.	1/512 Fract.	1/1024 Fract.
	32				Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.	1/512 Fract.
	64					Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.
	128						Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.

Design Builder Dialog Box - After Selecting Four Factors in 16 Experiments

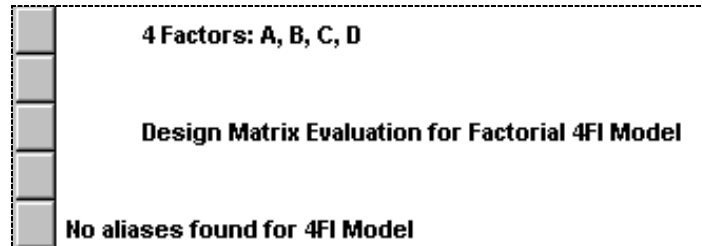
At the bottom of the design builder dialog box you see options to select the number of replicates of the design, the number of blocks and the number of centerpoints. Leave them at their defaults. If you want details on these design features, right click on the word to see the “What’s This” pop-up Help. For example, if you right click on “Replicates” you get the “What’s This” option as shown below. Click on the “What’s This” and you get the explanation shown.



Other Design Features Investigated via Context-Sensitive Help System

Help like this is available in many places in the software. Check it out!

Click on the **Continue** button. The next screen will provide information about factor aliasing, which will be a concern only if you chose a fractional design (the non-white squares in design builder).



Design Evaluation - Alias Structure (No aliases in this case)

Since this example is a full factorial, it does not have aliased terms, so proceed to the next step by clicking on **Continue**. (Note: at any point in the design building process you can press the Back button to view previous screens and change earlier entries.)

Enter Factor Descriptions

Enter names for the factors, units and levels in this dialog box. Use the arrow keys, tab key or mouse to move from one space to the next. The Tab (or Shift Tab) key moves the cursor forward (or backward) between entry fields.

Factors can be of two distinct types - “Numeric” or “Categorical”. You change the type of factor by clicking on cells in the Type column and choosing “Categorical” from the drop down list, or by typing “N” for Numeric and “C” for Categorical. Numeric data comes from a continuous scale such as temperature or pressure. Categorical data, such as catalyst type or automobile model, occurs in distinct levels. Design-Expert permits characters for the levels of categorical factors. Leave the default as “Numeric” for all factors in this case.

	Name	Units	Type	Low	High
	Temperature	deg C	Numeric	24	35
	Pressure	psig	Numeric	10	15
	Concentration	percent	Numeric	2	4
	Stir Rate	rpm	Numeric	15	30

Factor Entry - After Entering Names and Levels

Enter the above names, units, low and high levels for the factors to complete the form. Then click on **Continue**.

Enter Response Description

The **Responses** dialog box appears after accepting the factor names and levels. Using the list arrow, you can enter up to 99 responses. In this case we only need to enter a single response name (Filtration Rate) and units (gallons/hour) as shown below.

Responses:	1
Name	Units
Filtration Rate	gallons/hr

Edited Response Values

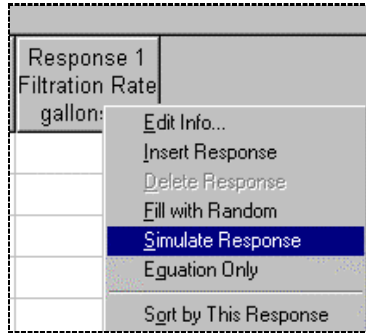
Click on **Continue** to accept these inputs and generate the design layout window.

Save the Design

When you complete the design setup, save it to a file by selecting **Save As** from the **File** menu. Type in the name of your choice (such as **Factorial.dx6**) for your data file. Then click on **Save**.

Enter the Response Data

At this stage you normally would print the run sheet, perform the experiments and record the responses. The software automatically lists the runs in randomized order, which protects against any lurking factors such as time, temperature, humidity or the like. To avoid the tedium of typing numbers, yet preserve a real-life flavor for this exercise, simulate the data by right-clicking the header of the **Response** column and selecting **Simulate Response**.



Simulating the Response for Filtration Rate

You will now see a list of “sim” files. Click on **Filtrate.sim** and **Open**. The simulation of the filtration process now generates the response data. Right click on the **Std** column header (on the gray square labeled Std) and select **Sort by Standard Order**. Your data will now match the tutorial except for a different random run order. (When doing your own experiments, always do them in random order. Otherwise, lurking factors that change with time will bias your results.)

	Std	Run	Block	Factor 1 A: Temperature deg C	Factor 2 B: Pressure psig	Factor 3 C: Concentration percent	Factor 4 D: Stir Rate rpm	Response 1 Filtration Rate gallons/hr	
		1	12	Block 1	24.00	10.00	2.00	15.00	45
		2	7	Block 1	35.00	10.00	2.00	15.00	71
		3	8	Block 1	24.00	15.00	2.00	15.00	48
		4	13	Block 1	35.00	15.00	2.00	15.00	65
		5	16	Block 1	24.00	10.00	4.00	15.00	68
		6	14	Block 1	35.00	10.00	4.00	15.00	60
		7	9	Block 1	24.00	15.00	4.00	15.00	80
		8	11	Block 1	35.00	15.00	4.00	15.00	65
		9	2	Block 1	24.00	10.00	2.00	30.00	43
		10	10	Block 1	35.00	10.00	2.00	30.00	100
		11	1	Block 1	24.00	15.00	2.00	30.00	45
		12	3	Block 1	35.00	15.00	2.00	30.00	104
		13	4	Block 1	24.00	10.00	4.00	30.00	75
		14	6	Block 1	35.00	10.00	4.00	30.00	86
		15	5	Block 1	24.00	15.00	4.00	30.00	70
		16	15	Block 1	35.00	15.00	4.00	30.00	96

Design Layout in Standard Order - Data Entered

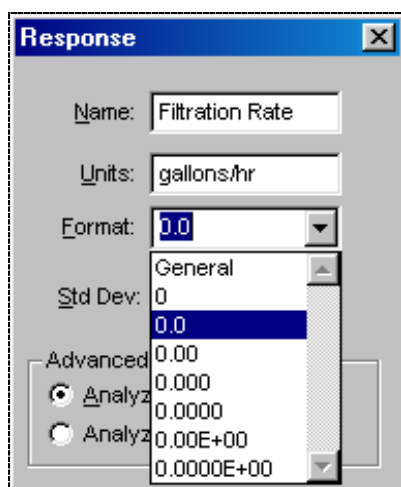
Do a **File, Save** to save your work.

The response data comes in under a general format. You will get cleaner outputs if you change this to a fixed format. Put your mouse over the response column heading (top of the response column) and do a right click.

Factor 1	Factor 4 D: Stir Rate rpm	Response 1 Filtration Rate gal/hr
2.00	15.00	45
2.00	15.00	
2.00	15.00	
2.00	15.00	
4.00	15.00	
4.00	15.00	60
4.00	15.00	80
4.00	15.00	65
2.00	30.00	43

Menu for Manipulating Response Column - Available with a Right Mouse Click

As you can see, Design-Expert offers many options on this menu. Select **E**dit Info to get a dialog box for format change. Choose the **F**ormat 0.0 and then press the **O**K button.



Changing the Format of a Response

You can change the format on the input factors also, or change names or levels. Check this out by doing a right mouse click on any of the other column headings. Then continue with the tutorial.

Design-Expert provides two methods of displaying the levels of the factors in a design:

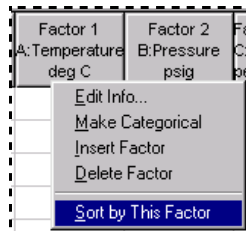
- Actual levels of the factors.
- Coded as -1 for low levels and +1 for high levels.

The default design layout is actual factor levels in run order. To look at the design in coded values, click on **D**isplay **O**ptions from the menu bar and select **P**rocess **F**actors - **C**oded. Your screen should now look like the one shown below.

		Process Factors		Actual					
		Mixture Components		✓ Coded					
		Responses in Original Scale	Responses in Transformed Scale	Factor 1	Factor 2	Factor 3	Factor 4	Response 1	
				A: Temperature	B: Pressure	C: Concentration	D: Stir Rate	Filtration Rate	
				deg C	psig	percent	rpm	gallons/hr	
1	12	{ 1 }	-1.000	-1.000	-1.000	-1.000	-1.000	45.0	
2	7	{ 1 }	1.000	-1.000	-1.000	-1.000	-1.000	71.0	
3	8	{ 1 }	-1.000	1.000	-1.000	-1.000	-1.000	48.0	
4	13	{ 1 }	1.000	1.000	-1.000	-1.000	-1.000	65.0	
5	16	{ 1 }	-1.000	-1.000	1.000	-1.000	-1.000	68.0	
6	14	{ 1 }	1.000	-1.000	1.000	-1.000	-1.000	60.0	

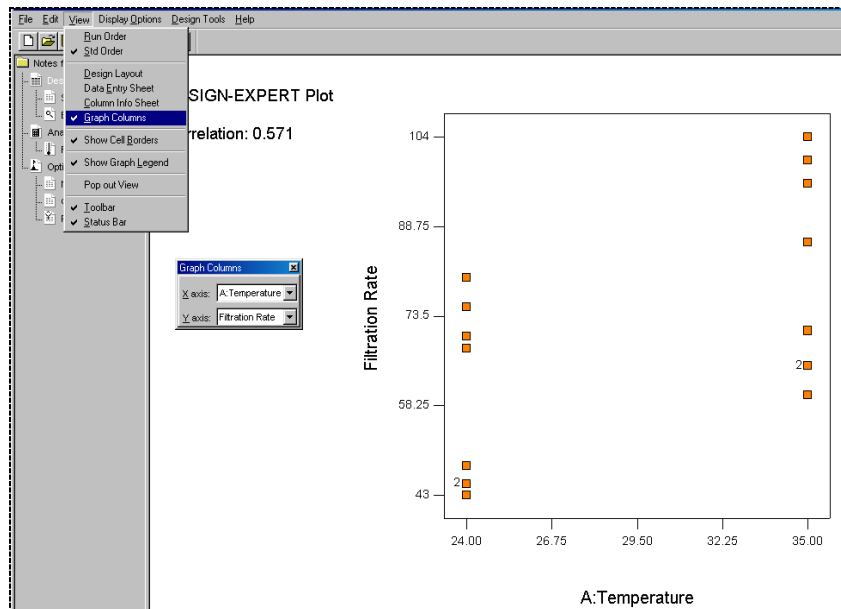
Design Layout - Coded Factor Levels

Design-Expert provides various ways for you to visualize your data before moving on to an in-depth analysis. For example, you can sort the design by any column. Convert the factors back to their original values by clicking on **Display Options** from the menu bar and selecting **Process Factors - Actual**. Move your mouse to the top of column **Factor 1** and click the right mouse button. Then select **Sort by This Factor**.



Sorting the Design on a Factor

You will now see more clearly the impact of temperature on the response. Better yet, you can make a plot of the response versus factor A by selecting **View, Graph Columns** from the menu. Confirm that the **X-axis** is set at **A: Temperature** and the **Y-axis** at **Filtration Rate** by clicking on the drop down lists for the x-axis and y-axis to produce the plot shown below. You can see that temperature makes a big impact on the response. If you like, go ahead and look at other factors versus the response.



Graph of Factor Temperature versus Response Filtration Rate

The number “2” appears besides a few points on this plot. This notation indicates the presence of multiple points at the same location. Click on one of these points more than once to identify the individual runs. After you are done exploring, return to the design layout screen by selecting **View, Design Layout** from the menu. Next you will do a statistical analysis to find out what’s really going on.

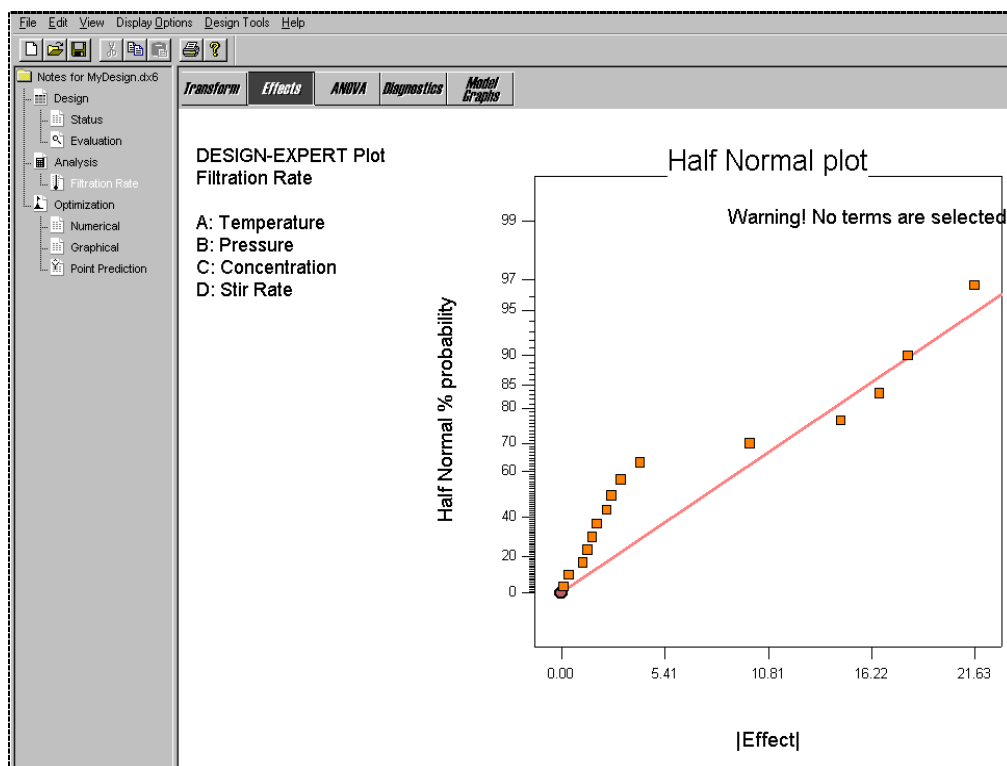
Analyze the Results

To begin analyzing the design, click on the **Filtration Rate** response node on the left side of your screen. This brings up the analytical tool bar across the top of the screen. To do the statistical analysis, just click the buttons progressively from left to right.

The **Transform** button is initially highlighted. It displays a list of mathematical functions that you may apply to your response. We described this earlier in the section on one-factor designs. In this case you can leave the screen at its default: **None**.

View Effects, Select the Vital Few Significant Ones

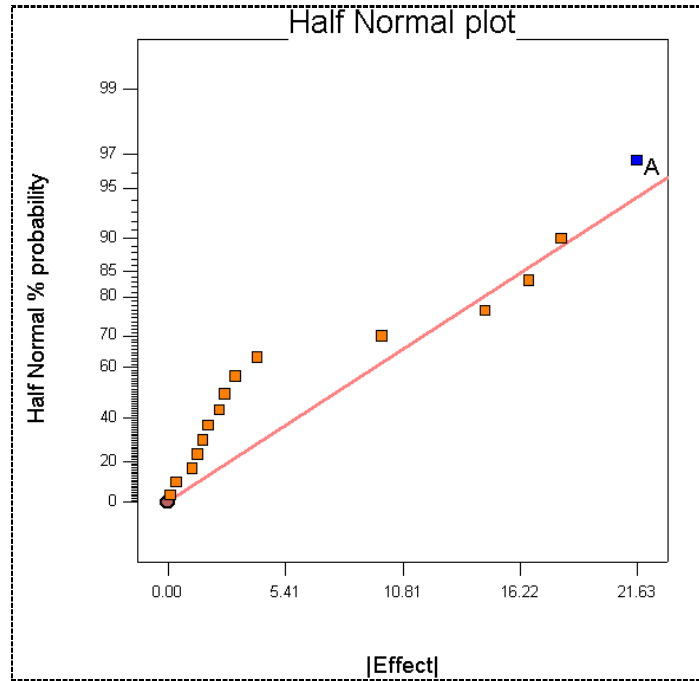
Click on the **Effects** button. Design-Expert displays the absolute value of all effects (plotted as squares) on a half-normal probability plot.



Half-Normal Plot of Effects with Nothing Selected

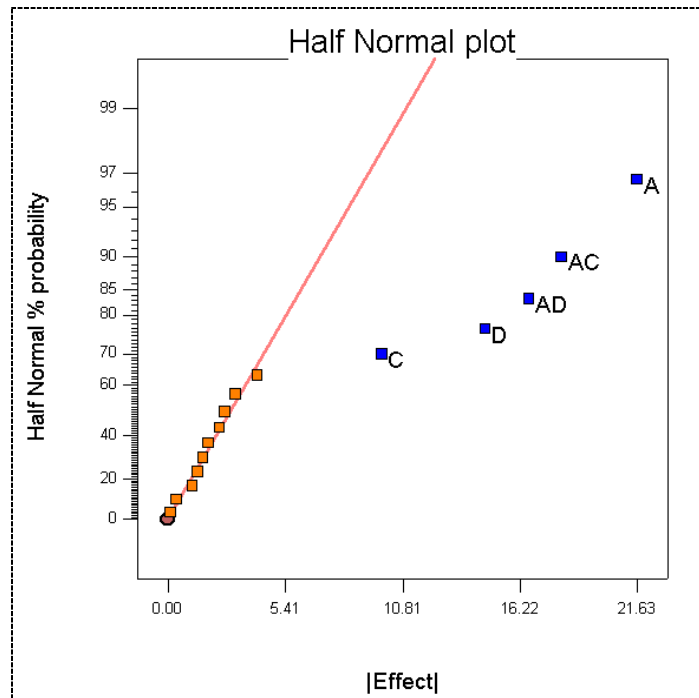
Note the warning message on the screen: “No terms are selected.” You must choose which effects to include in the model. If you try to proceed at this point, you will get another warning that “you have not selected any factors for the model.” The program will allow you to press ahead with only the mean as the model (no effects), or you can opt to be sent back to the Effects view (a much better choice!).

You can select effects by simply clicking on the square points. Start with the largest effect at the right side of the plot.



First Effect Chosen

Keep picking from right to left until the line matches up with the majority of the effects near zero. Notice that Design-Expert adjusts the line to exclude the chosen effects. You should see a noticeable gap between the line and the next effect (the smallest one chosen).



Half-Normal Probability Plot – All Big Effects Selected

If you accidentally select an effect, just click it again to deselect it. Also, you can adjust the line by dragging the end with the mouse. (Click and hold down the left mouse button to drag.)

Use the **View, Effects List** selection of the menu bar to present a numerical table of the results.

The screenshot shows the Design-Expert 6 software interface. The 'View' menu is open, and 'Effects List' is selected. The main window displays a table of results under the 'Effects' tab. The table has columns for Term, Stdized Effects, Sum of Squares, and % Contribution. The effects are listed in descending order of Sum of Squares. Selected effects are marked with a green 'M', and unselected effects are marked with a blue 'e'.

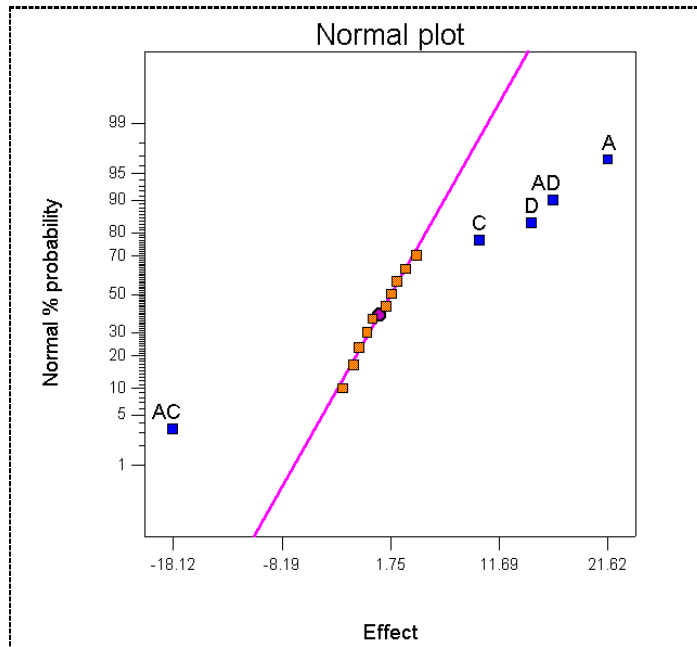
Term	Stdized Effects	Sum of Squares	% Contribution
Intercept			
M A	21.63	1870.56	32.64
e B	3.13	39.06	0.68
e C	9.88	390.06	6.81
M D	14.63	855.56	14.93
e AB	0.12	0.063	1.091E-003
M AC	-18.12	1314.06	22.93
M AD	16.63	1105.56	19.29
e BC	2.38	22.56	0.39
e BD	-0.38	0.56	9.815E-003
e CD	-1.13	5.06	0.088
e ABC	1.88	14.06	0.25
e ABD	4.13	68.06	1.19
e ACD	-1.63	10.56	0.18
e BCD	-2.62	27.56	0.48
e ABCD	1.37	7.56	0.13
Lenth's ME	6.75		
Lenth's SME	13.70		

Effects List

The table lists all the estimable effects for the coded levels of the factors. The chosen effects are designated by an “M”, indicating their selection for the predictive model (M stands for model). Unchosen effects are labeled “e” for error. These effects will be incorporated in the residual used to test the model in the subsequent ANOVA.

The software also reports the Lenth’s margin of error (ME) and simultaneous margin of error (SME). Notice that the View menu lets you select by these criteria. We recommend against this because Lenth’s method doesn’t pick effects in a consistently good manner. The SME criteria is more conservative and won’t pick as many effects as the ME criteria. It may work better for larger data sets (at least 32 runs.) (For details, see Lenth, “Quick and Easy Analysis of Unreplicated Factorials” in *Technometrics*, November 1989, Volume 31, Number 4, page 169.) If you do use one of Lenth’s criteria for selecting effects, be sure to look at the results on the half-normal graph and be prepared to over-ride the selection. Remember that you can click on any given effect to select or deselect it for modeling purposes.

Design-Expert offers a view of the effects on a full-normal plot also. Select **View, Normal Graph** to check this out. The full-normal probability plot is less sensitive for selecting small effects, but it does give you the differentiation between positive and negative effects. Notice the round point at the center of the line. This acts as a handle for you to move the line, or a pivot point, around which you can rotate the line. The pivot point can be moved along the line. You are in control: Check it out!



Normal Probability Graph

ANOVA and Statistical Analysis

It is now time to look at the statistics in detail with the analysis of variance (ANOVA) table. Click on the **ANOVA** button to see the selected effects and their coefficients. You already did this once when you analyzed the one-factor data in the previous section. The approach for two-level factorials will be much the same. There are two views of the ANOVA results. Choose **View, Annotated ANOVA** for the analysis of variance information, with text to guide your interpretation.

Transform	Effects	ANOVA	Diagnostics	Model Graphs		
Use your mouse to right click on individual cells for definitions.						
Response: Filtration Rate						
ANOVA for Selected Factorial Model						
Analysis of variance table [Partial sum of squares]						
Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	5535.81	5	1107.16	56.74	< 0.0001	significant
A	1870.56	1	1870.56	95.86	< 0.0001	
C	390.06	1	390.06	19.99	0.0012	
D	855.56	1	855.56	43.85	< 0.0001	
AC	1314.06	1	1314.06	67.34	< 0.0001	
AD	1105.56	1	1105.56	56.66	< 0.0001	
Residual	195.12	10	19.51			
Cor Total	5730.94	15				
The Model F-value of 56.74 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise.						
Values of "Prob > F" less than 0.0500 indicate model terms are significant.						
In this case A, C, D, AC, AD are significant model terms.						

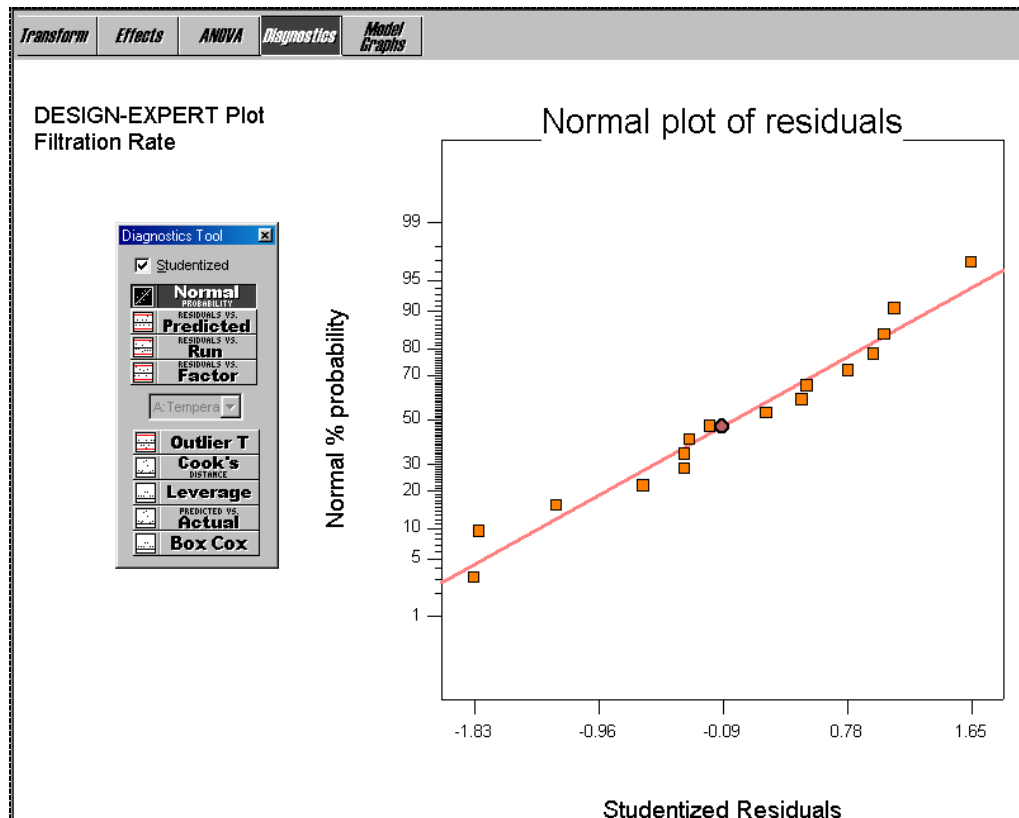
ANOVA Report – Annotated View

Check the probability (“Prob>F”) for the Model. As a general rule, values of 0.05 or less indicate significance. Also inspect the model terms, in this case: A, C, D, AC, AD. Check the associated F-tests (“Prob>F”) for significance. In this case, all the terms pass the 0.05 test with room to spare.

Pop-up definitions for numbers on the ANOVA table can be obtained by right clicking and choosing **Help**. Give this a try! When you finish with Help, close it via the **File, Exit** menu option. Further down in the ANOVA output you will find predictive equations both in coded and uncoded form, as well as a case table of actual values, predicted values, residuals and calculated statistics for use in validating the model. Rather than dwell on all these statistics, many of which have already been described for the one-factor case, let’s look at some graphs that will be more helpful for the diagnostics.

Validate the Model

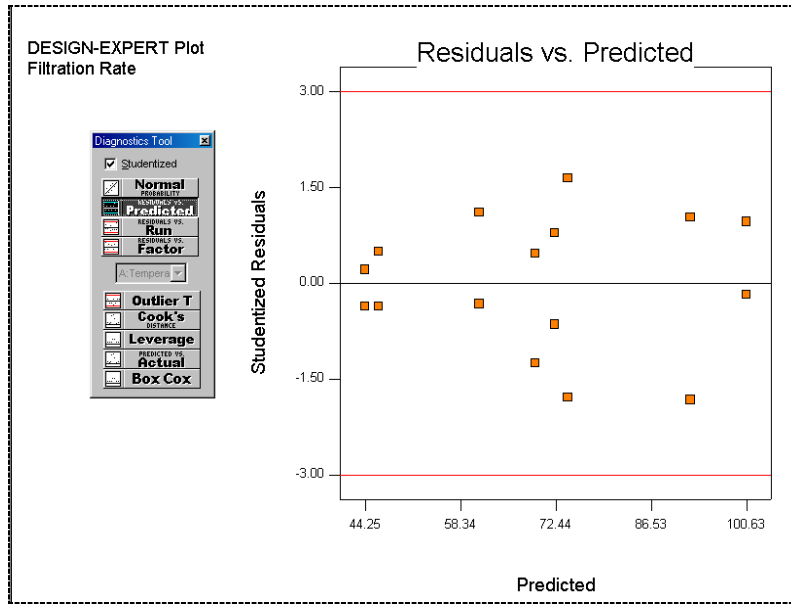
Click on the **Diagnostics** button. The default shows a **Normal Probability** plot of the studentized residuals. (Number of standard deviations of the actual values from their respective predicted values).



Normal Probability of Studentized Residuals

Ideally this will be a straight line, indicating no abnormalities. In this case the plot looks OK.

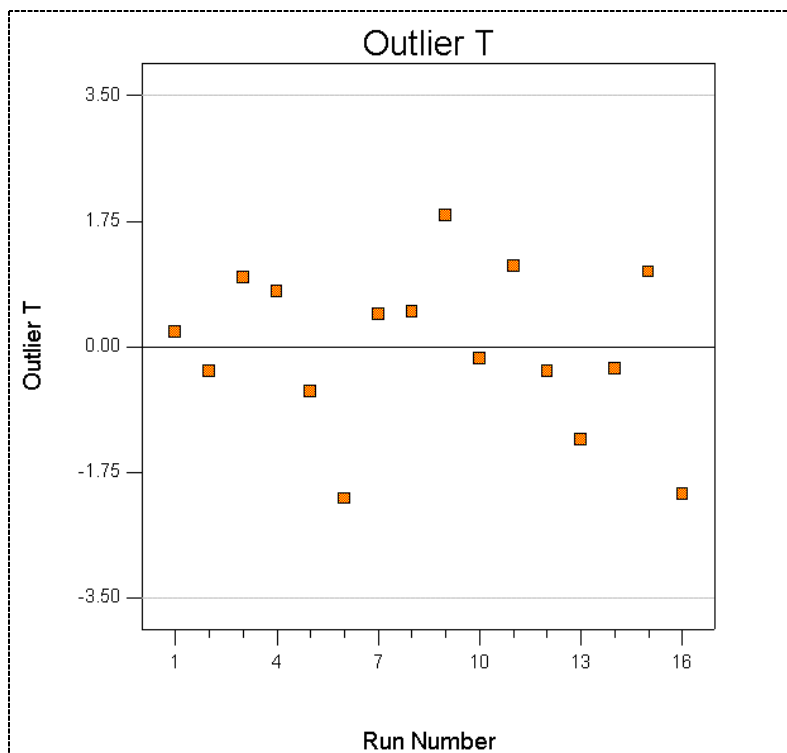
Select **Residuals vs. Predicted** from the **Diagnostic Tools** palette.



Studentized Residuals versus Predicted Values

The size of the studentized residual should be independent of its predicted value. In other words the spread of the studentized residuals should be approximately the same across all levels of the predicted values. In this case the plot looks OK.

Select **Outlier T** from the **Diagnostics Tool** palette to see if any points stand out.

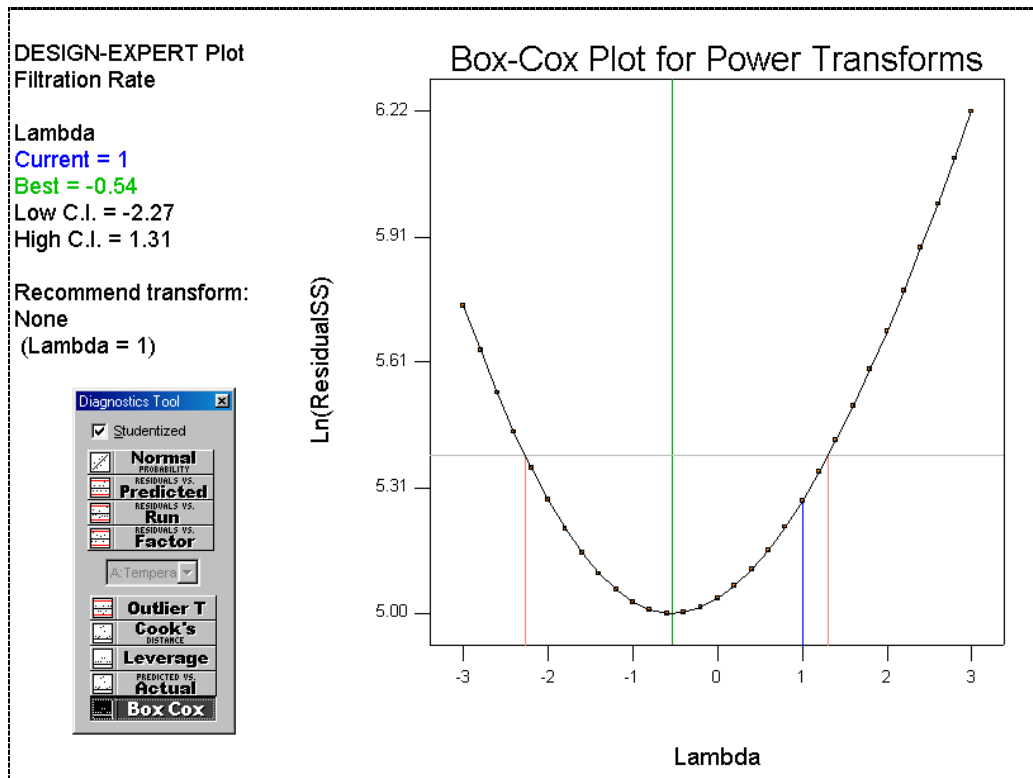


Outlier T Plot (Note: your graph may differ due to random run order)

Since this graph is plotted in randomized run order, the ordering of the points on your screen will be different than shown here. But we're not looking for patterns, just

outliers. There's nothing out of the ordinary here - all the points fall well within the red lines set at plus-or-minus 3.5. However, if there were an outlier, you could click on it to get the coordinates displayed to the left of the graph. The program remembers the point. It will remain highlighted on other plots. This is especially helpful in the residual analysis, because you can track a suspect point. This feature also works in the interpretation graphs. Give it a try! Click anywhere else on the graph to turn the point off.

The last diagnostic plot on the tool palette is the Box Cox plot. This was developed to analytically calculate the best power law transformation. (Refer to Montgomery's book for details.) Select **Box Cox** from the tool palette.



Box Cox Plot for Power Transformations

The text on the left side of the screen gives the recommended transformation, in this case "None." That's all you really need to know!

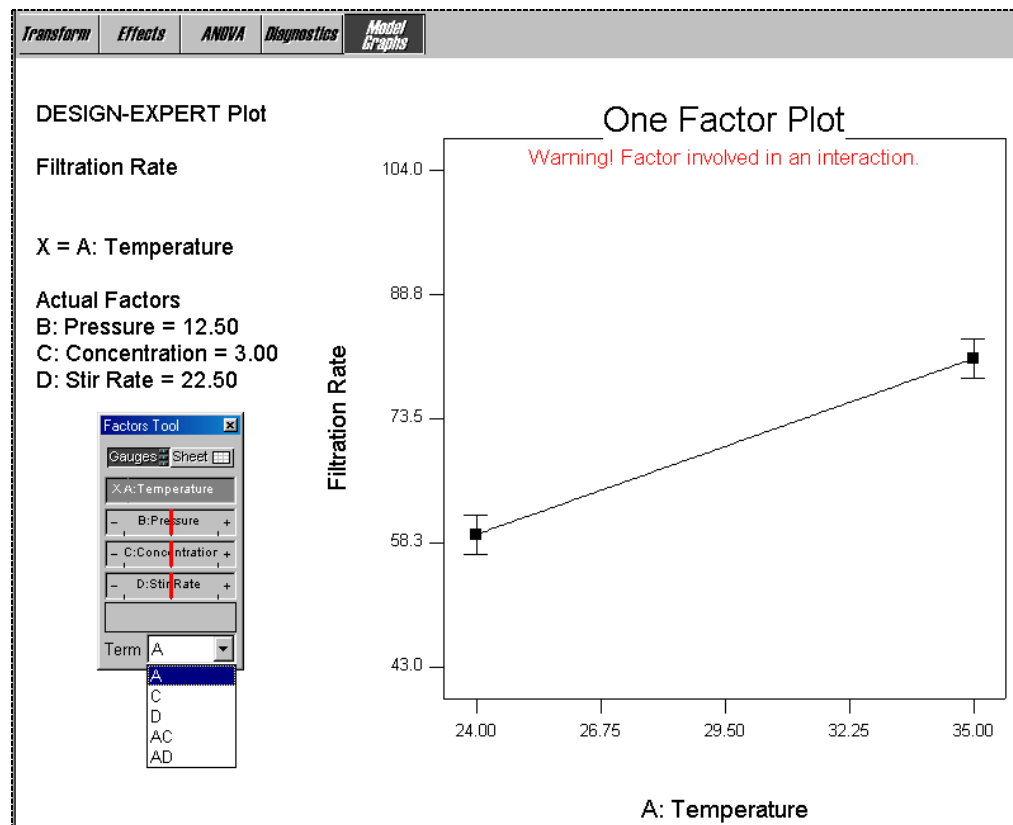
For those of you who want to dig into the details, note that the Box-Cox screen is color coded to help with interpretation. The blue line shows the current transformation. In this case it points to a value of 1 for "Lambda," which symbolizes the power applied to your response values. A lambda of 1 indicates no transformation. The green line indicates the best lambda value, while the red lines indicate the 95% confidence interval surrounding it. If this 95% confidence interval includes 1, then no transformation is recommended.

The Box-Cox plot will not help if the appropriate transformation is either the logit or the arcsine square root transformation. See the Help system for further details on when these transformations would be appropriate.

Examine Main Effects and Any Interactions

Assuming that the residual analyses do not reveal any problems (no problems are evident in our example), it's now time to look at the significant factor effects.

On the analytical tool bar at the top of the screen, choose the **Model Graphs** button. The floating **Factors Tool** palette will open along with the default plot. Move the floating palette as needed by clicking on the top blue border and dragging it. This tool controls which factor(s) are plotted on the graph. At the bottom of the Factors Tool is a pull-down list from which you can also select the factors to plot. Only the terms that are in the model are included in this list. If you select a single factor (such as A) the graph will change to a One Factor Plot. (You can also choose one factor plots via the View menu). If you choose a two-factor interaction term (such as AC) the plot will become the interaction graph of that pair. (Note: you can also choose interaction graphs via the View menu.) Click on the list arrow and select the effect of **A**.

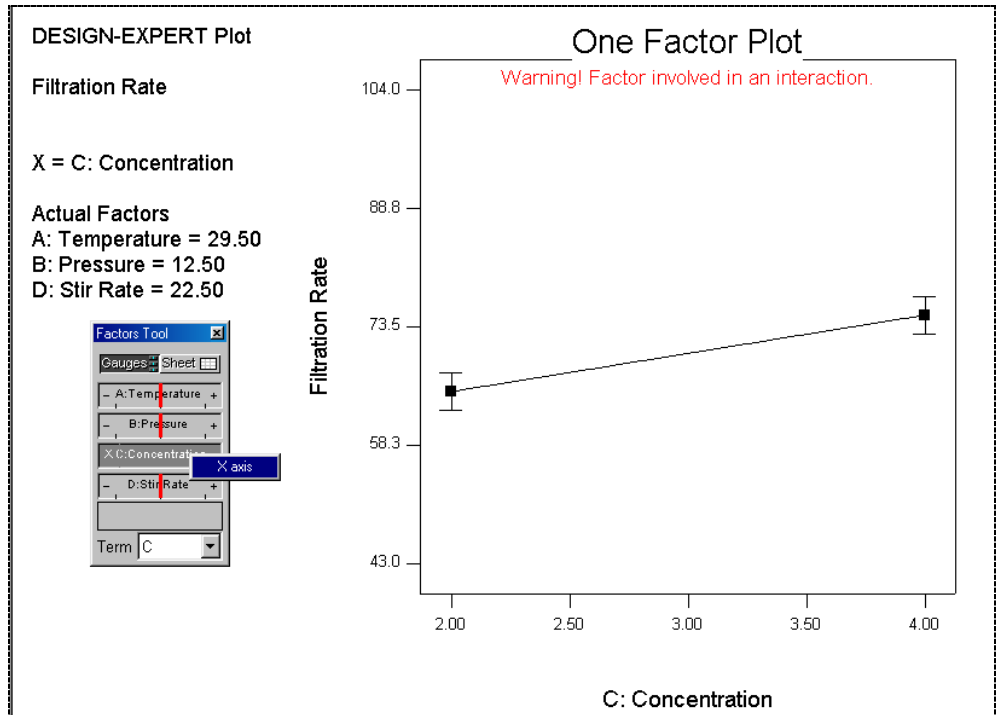


Factors Tool – Gauges View

(If your graph is displayed in coded units, return to actual units by choosing Display Options, Process Factors – Actual.)

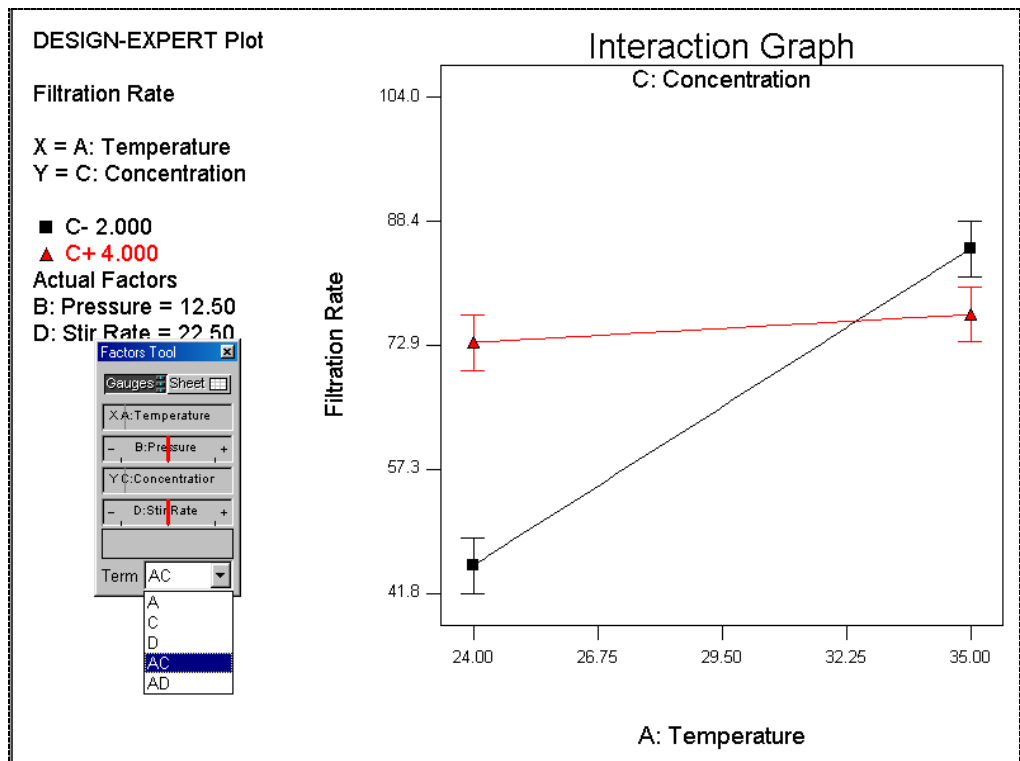
Note the warning at the top of the plot of A (Temperature). It says “Factor involved in an interaction.” You should never try to interpret main effects plots of factors involved in interactions, because it will provide misleading information.

Let's try another main effect by right-clicking over the factor **C** (Concentration) and changing it to the **X-Axis**.



Changing the X-axis on the Factors Tool

The same warning pops up, because C participates with A in the AC interaction. Click on the list arrow and select the effect of interaction **AC**, which the program originally displayed by default (smart software!).

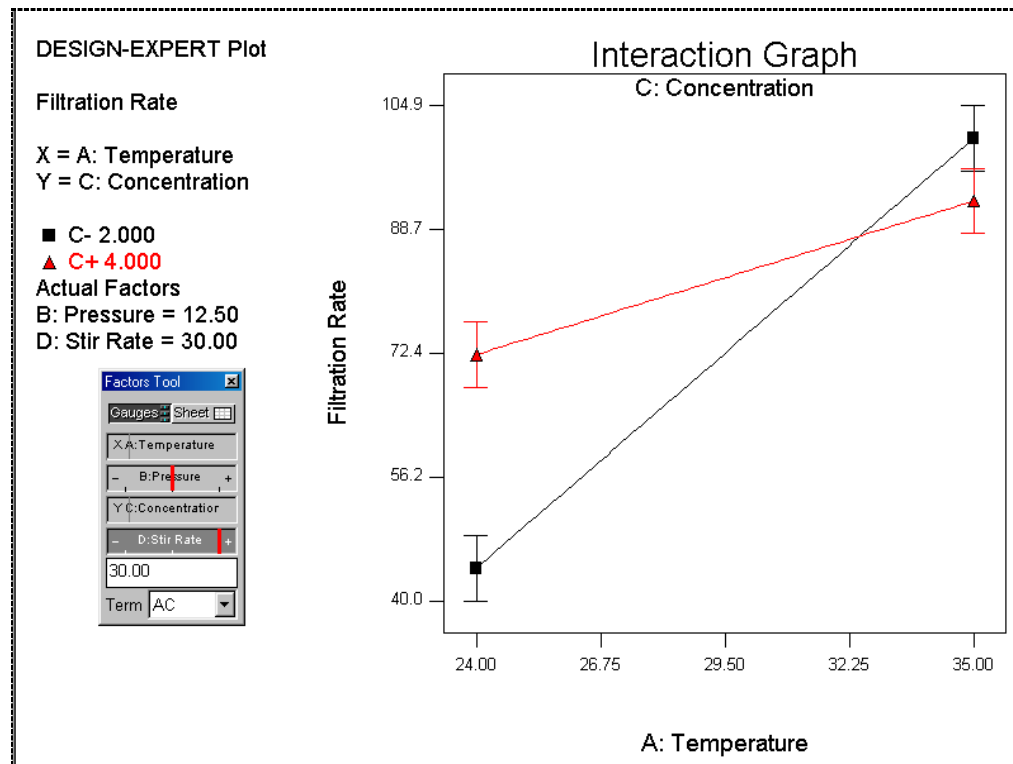


Model Graph -The AC Interaction

The “I-Beam” symbols on this plot (and other effect plots) depict the 95% least significant difference (LSD) interval for the plotted points. To get a numerical value for LSD, click on any of the points (squares or triangles representing predicted outcome). Try it!

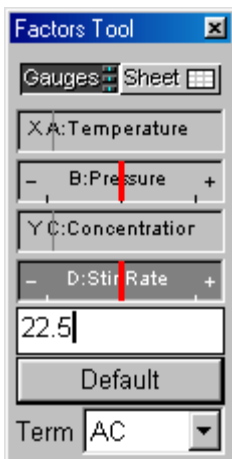
Those points that have non-overlapping intervals are significantly different. In this case the spread of the points on the right side of the graph (where Temperature is high) is smaller than the spread between the points at the left side of the graph (where Temperature is low.) In other words, the effect of formaldehyde concentration (C) is less significant at the high level of temperature (A). Therefore, the experimenters can go to high temperature and reduce the concentration of harmful formaldehyde, while maintaining or even increasing filtration rate.

Notice on the Factors Tool that factors not already assigned to an axis (X or Y) display a red slider bar, which allows you to choose specific settings. The bars default to the midpoint levels of these non-axes factors. You can change their levels by dragging the bars, or by typing the desired level in the numeric space near the bottom of the Factors Tool. Check this out by grabbing the slide bar for factor **D** (Stir Rate) and moving it left and right. Notice that the interaction graph changes.



Interaction AC with the Slider Bar for Factor D Set Far Right at High (+) Level

To reset the graph to its default, type **22.5** at the bottom of the Factors Tool. (Factor D must be clicked and highlighted for this to take effect.) You can also get the original settings back by pressing the Default button, a relatively new feature not shown on all the screen shots in this manual. Give it a try!



Re-setting D (Stir Rate) Back to Centerpoint Level by Entering Desired Value (22.5)

Now, switch to the Sheet View by clicking on the **Sheet** button on the Factors Tool.

	Factor	Axis	Value	Axis Low	Axis High
A	Temperature	X		24.00	35.00
B	Pressure		12.50	10.00	15.00
C	Concentration	Y		2.00	4.00
D	Stir Rate		22.50	15.00	30.00

Sheet View of Factors Tool

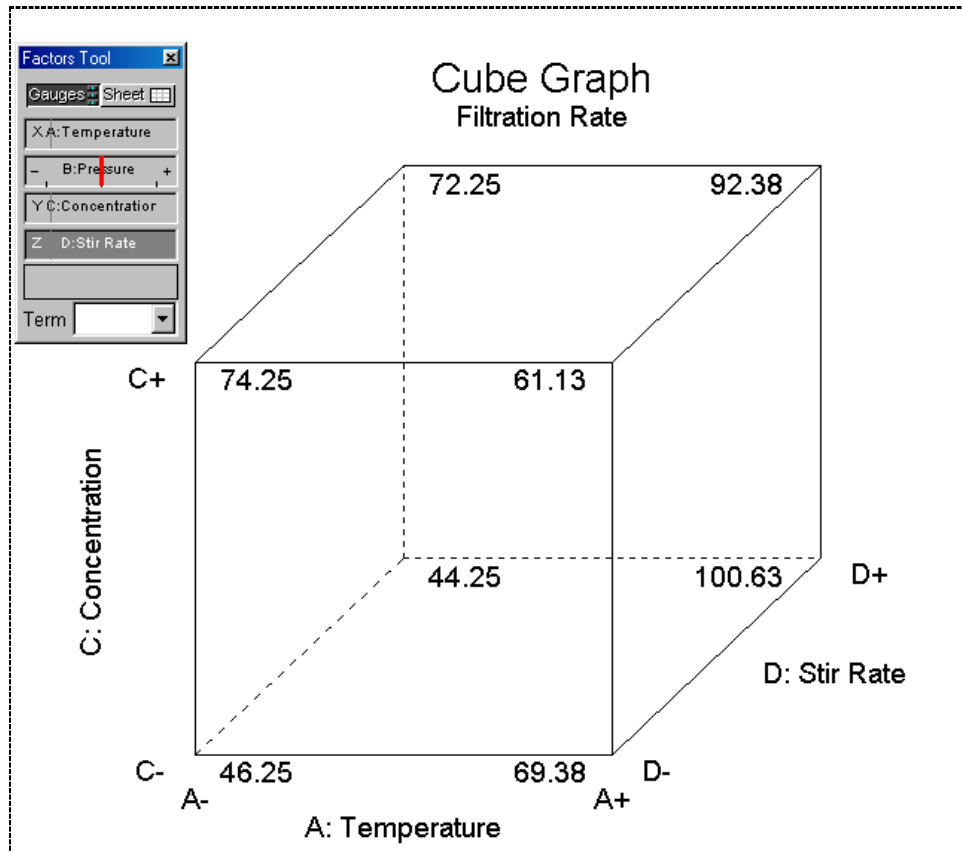
This view offers alternative modes for specifying how to set up your plot. In the columns labeled “Axis” and “Value” you can change the axes settings or type in specific values for factors. Now return to the Gauges view by clicking on the **Gauges** button.

You can view the AC interaction with the axes reversed by right clicking on either factor (A or C) on the Factors Tool and putting it on the other axis (X or Y). Check it out! It makes no difference statistically.

You can edit at least some text on many of the graphs by doing a right mouse click. For example, on the interaction graph you can right-click on the X-axis label. Then choose Edit Text. The program then provides an entry field. Try it! You can also right-click anywhere on the plot to get options for Graph Preferences. There you can change the layout of the X or Y axis, and make other changes. Check this out!

Draw the Cube Plot

Now click on **View, Cube**.



Cube Plot of A, C, and D, with View Menu, Show Legend Option Off

This plot shows how three factors combine to affect the response. All values shown are predicted values, which allows plots to be made even with missing actual data. Since the factors of interest here are A, C, and D, the program picked them by default. (You can change the axes by right-clicking any particular factor on the Factor Tool.) For a cleaner look, click off the legend by selecting **View, Show Legend**.

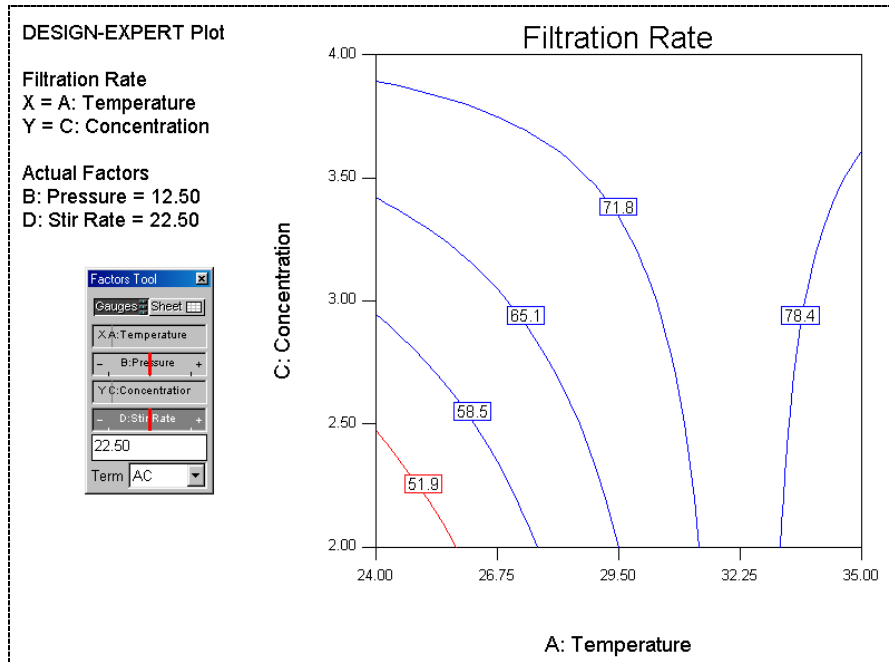
Filtration rate is maximum at the A+, D+, C- settings (lower back right corner), which also corresponds to the reduced formaldehyde concentration. Fantastic!!

Produce Contour and 3D Plots of the Interaction

An interaction represents a non-linear response of second order. It may be helpful to look at contour and 3D views of the interaction to get a feel for the non-linearity.

First select **View, Contour** to get a contour graph. Also, if you haven't done it already, turn the legend back on by choosing **View, Show Legend** (the checkmark appears when the legend is on.) The axes should come up as A (Temperature) and C (Concentration). If not, just right click over the Factor Tool and make the appropriate changes.

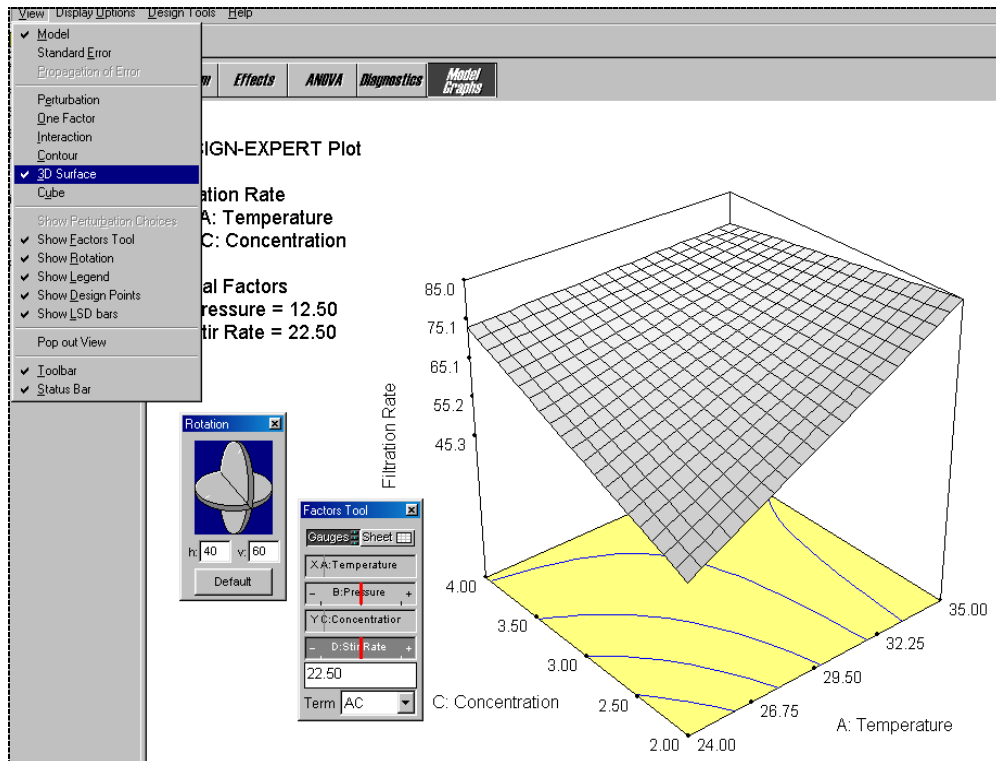
Design-Expert contour plots are highly interactive. For example you can click on a contour to highlight it. Then you can drag it to a new location. Right-click to add a contour or set a flag. For a detailed example of these features see the Response Surface Tutorial.



Contour Graph, Show Legend Back On

By placing the mouse over the graph and right clicking, you bring up a variety of options that are covered in depth in the Response Surface Tutorial. Explore the options if you wish and then move on.

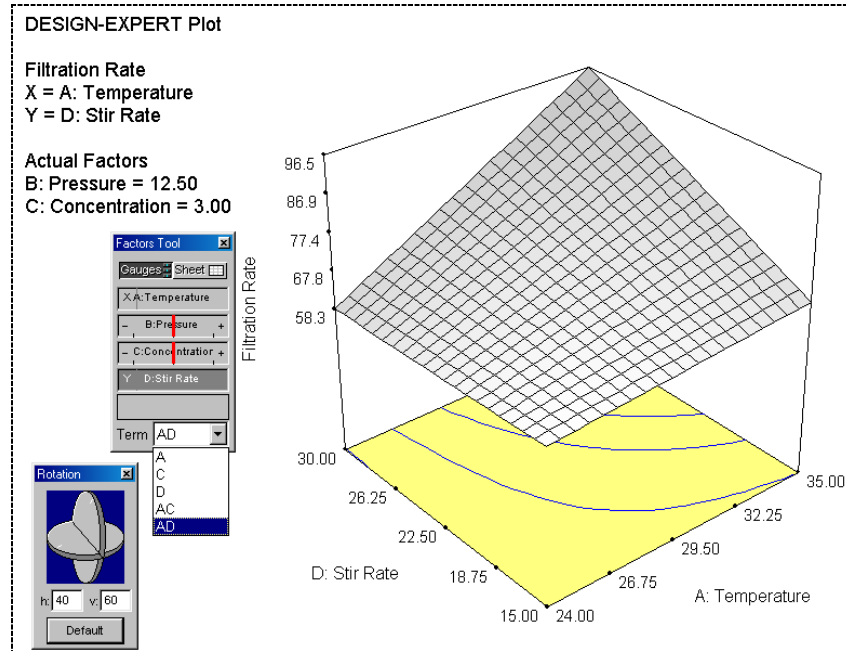
Now select **View, 3D Surface** from the menu.



A 3D View of the AC Interaction

Notice that there is now a **Rotation** tool. Grab the wheel with your mouse and rotate the plot to different angles. It's fun! Press the Default button to re-set the graph to its original coordinates.

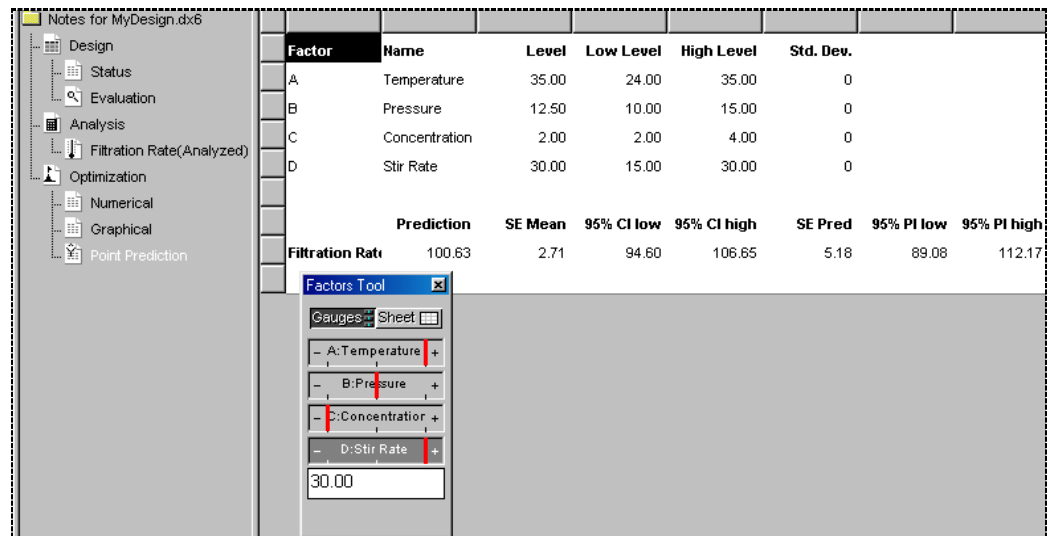
Before moving on to the last stage, take a look at the other interaction by clicking on the list arrow on the Factors Tool and selecting the effect of **AD**.



3D Plot for Second Interaction: AD

Point Prediction

The last feature that we will explore is the Point Prediction node. This is used to make predictions for your responses at any set of conditions. Click on the **Point Prediction** node located on the bottom of the list at the left side of your screen.



Point Prediction node with Best Settings

When you first enter this screen, the **Factors Tool** palette will be defaulting to the midpoints of each factor. You will need to use either the red slider bars or the Sheet view to set each factor to its desired level. For this tutorial, the analysis suggests that you should slide the factors as follows:

- **A** (Temperature) right to its high level **(+)**
- **B** (Pressure) leave at default level of centerpoint
- **C** (Concentration) left to its low level **(-)**
- **D** (Stir Rate) right to its high level **(+)**

This will provide the highest predicted filtration rate with the least amount of formaldehyde.

Besides the response prediction value, several other pieces of information are provided. The “SE Mean” is the standard deviation associated with the prediction of an average value at these settings. The “95% CI” is the confidence interval that is calculated to contain the true mean 95% of the time. The “SE Pred” is the standard deviation associated with the prediction of an individual observation. Finally, the “95% PI” is the prediction interval calculated to contain the true value of an individual observation 95% of the time. All of these values can be used to manage expectations of the process. Note that the 95% confidence interval on a mean will be a narrower spread than the 95% prediction interval for a single observation.

You’ve now viewed all the important outputs for analysis of factorials. We suggest you do a **File, Save** at this point to preserve your work. Design-Expert will save the model you created so the outputs can be quickly reproduced if necessary. If you want to put some comments on the file for future identification, click on the **Notes** folder node at the top left of the tree structure at the left of your screen. Then type in the description. It will be there to see when you re-open the file in the future.



Attaching a Comment to Your Data

Prepare Final Report

Now all that remains is to prepare and print the final reports. If you haven’t already done so, just click on the appropriate icon(s) and/or buttons to bring the information back up on your screen, and click on the print icon (or use the File, Print command).

You can also copy graphs to other applications: Use Edit, Copy. For ANOVA or other reports be sure to do a Select All first, or highlight the text you wish to copy.

This completes the basic tutorial on factorial design. Move on to the following advanced topic if you like, or exit from Design-Expert by choosing **File, Exit** from the menu. If you have not stored your data, or you made changes since the last save, a

warning message will appear. Exit only when you are sure that your data and results have been stored.

Fractional Factorial with Foldover (Advanced Topic)

At the outset of your experimental program you may be tempted to design one comprehensive experiment that includes all known factors - to get the BIG Picture in one shot. This assumes that you can identify all the important factors and their optimal levels. A more efficient, and less risky, approach consists of a sequence of smaller experiments. You can then assess results after each experiment and use what you learn for design of the next experiment. Factors may be dropped or added in mid-stream, and levels evolved to their optimal range.

Highly fractionated experiments make good building blocks for sequential experiments. Many people use Plackett-Burman designs for this purpose, but we prefer the standard two-level approach. Regardless of your approach, you may be confounded in the interpretation of effects from these low-resolution designs. Of particular concern, main effects may be aliased with plausible two-factor interactions. If this occurs, you might be able to eliminate the confounding by running further experiments using a foldover design. This technique adds further fractions to the original design matrix.

We will discuss foldover designs that offer the ability to:

- free main effects from two-factor interactions, or
- de-alias a main effect and all of its two-factor interactions from other main effects and two-factor interactions.

Design-Expert's "Foldover" feature will automatically generate the additional design points needed for either type of foldover.

Saturated Design Example

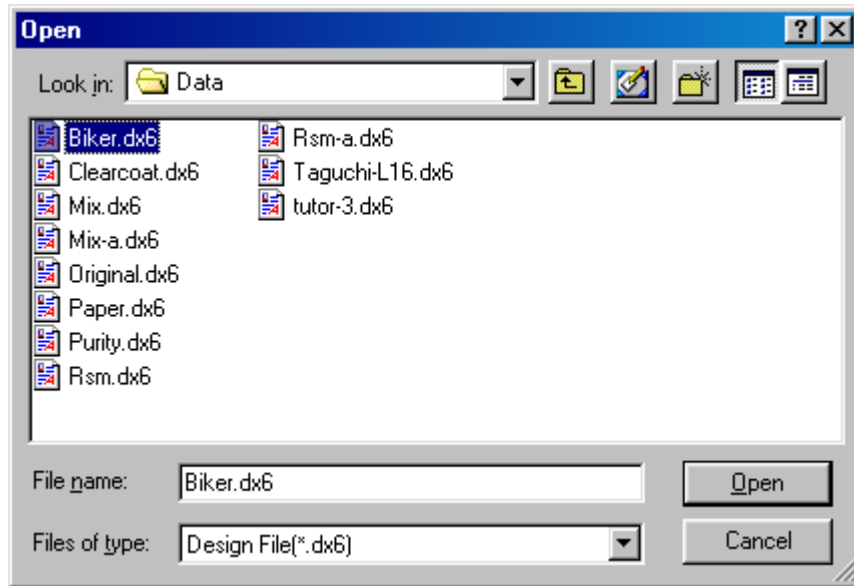
Dick has one week to fine-tune his bicycle before the early-bird Spring meet. He decides to test seven factors in only eight runs around the quarter-mile track! We show Dick's design below.

A	B	C	D	E	F	G	H
Seat	Tires, Psi	Handle - bars	Helmet Brand	Gear	Wheel Covers	Gene-rator	Time, Secs
Up	40	Up	Windy	Low	Off	Off	77
Down	40	Up	Atlas	High	Off	On	74
Up	50	Up	Atlas	Low	On	On	82
Down	50	Up	Windy	High	On	Off	47
Up	40	Down	Windy	High	On	On	72
Down	40	Down	Atlas	Low	On	Off	77
Up	50	Down	Atlas	High	Off	Off	48
Down	50	Down	Windy	Low	Off	On	83

Experimental Design and Results

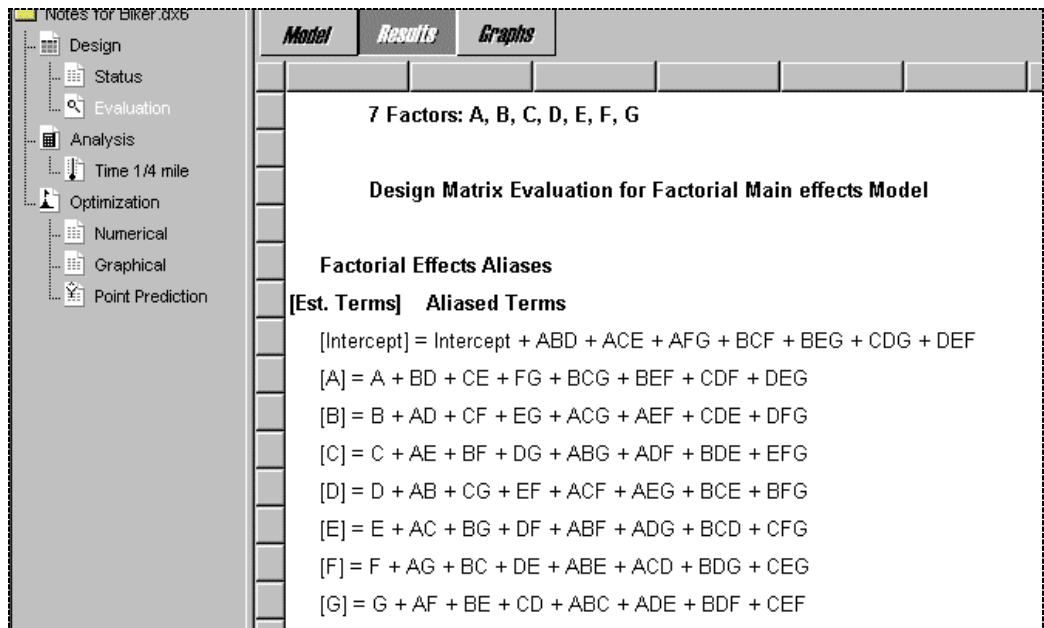
This is a Resolution III design in which all main effects are confounded with two-factor interactions. He would prefer to run a Resolution V or higher experiment, which would give clear estimates of main effects and their two-factor interactions. Unfortunately, this requires 64 experiments - too many for the short time remaining before competition.

To save time on data entry, select **File, Open Design** on the file **Biker.dx6**.



Opening the File of Biking Results

After looking over the data, check out the alias structure of the design by clicking on the **Evaluation** icon on the left and the **Results** icon at the top of the data window.

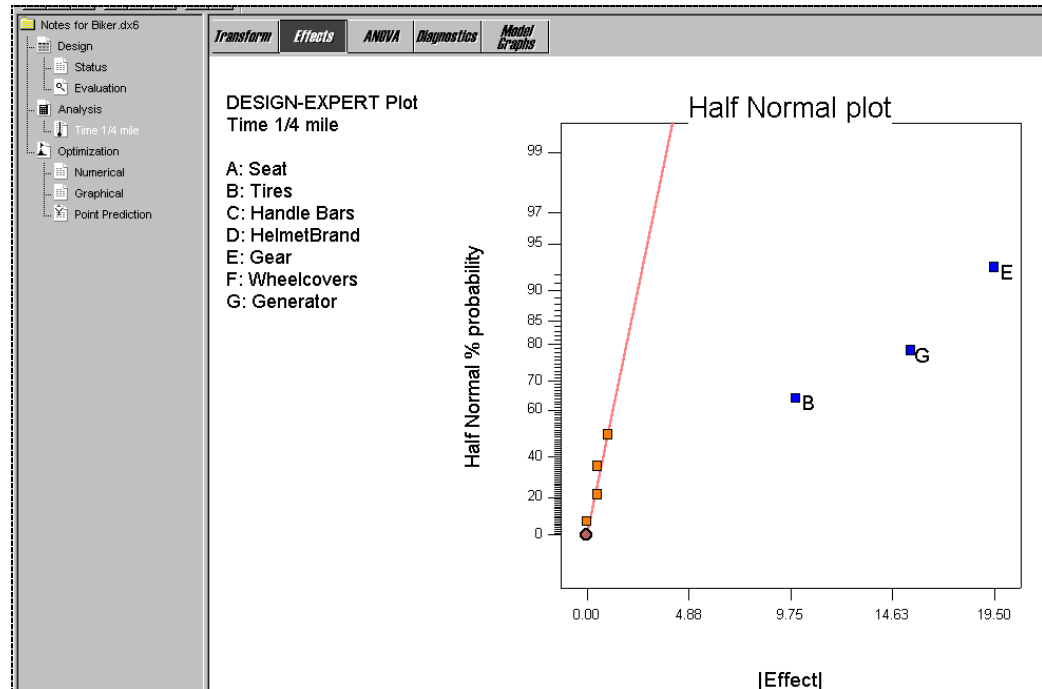


First Biking Trials - Alias Structure

This design is a 1/16th fraction, so every effect will be aliased with 15 other effects. (Note: to avoid unnecessary screen clutter, Design-Expert does not display interactions)

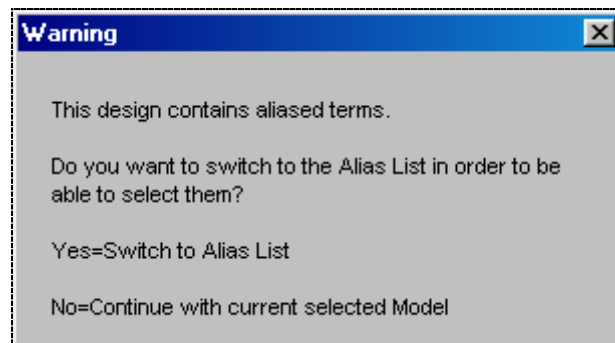
of four or more factors.) The output indicates that each main effect will be confounded with three two-factor interactions plus four three-factor interactions. The three-factor interactions can safely be ignored, but the two-factor interactions cannot.

Click on the **Time ¼ mile** node in the analysis branch at the left and **Effects** at the top of the display window. Starting from the right, pick the three largest effects. Notice how the line then jumps up and a gap appears between the trivial many effects near the zero effect level and the vital three effects that you picked.



Effect Graph

Factors B (Tires), E (Gear), and G (Generator) are clearly significant. Click the **ANOVA** icon to test this statistically. Before you can go there you are reminded that this design is aliased.



Alias Warning

You know about this from the design evaluation, so click on **No** to continue. (Option: click Yes to go back and see the list. Then click on ANOVA button.)

Transform	Effects	ANOVA	Diagnostics	Model Graphs		
Use your mouse to right click on individual cells for definitions.						
Response: Time 1/4 mile						
ANOVA for Selected Factorial Model						
Analysis of variance table [Partial sum of squares]						
Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	1441.00	3	480.33	640.44	< 0.0001	significant
B	200.00	1	200.00	266.67	< 0.0001	
E	760.50	1	760.50	1014.00	< 0.0001	
G	480.50	1	480.50	640.67	< 0.0001	
Residual	3.00	4	0.75			
Cor Total	1444.00	7				

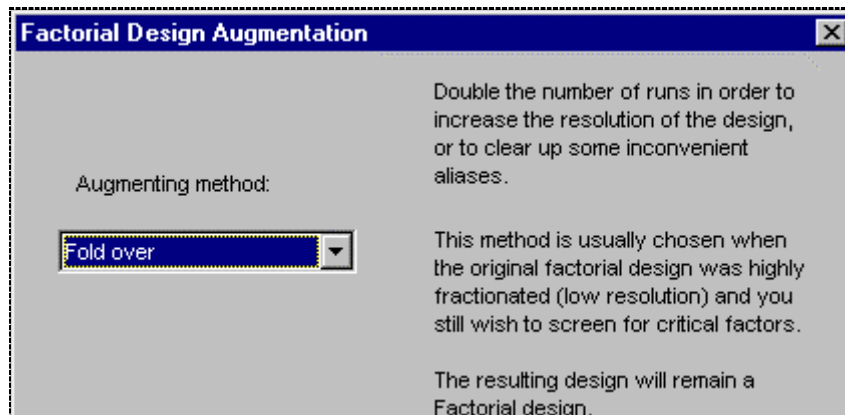
ANOVA Output

Dick gets a great fit to his data - it seems that hard tires decrease his time around the track, high gear gives him better speed and the generator should be left off. The whole approach looks very scientific and definitive. However, Dick's friend (and personal coach) Jane spots a fallacy in the conclusions. She points out that all of the main effects in this design are confounded with two-factor interactions. Maybe one of those confounded interactions is actually what's important. "You must run more experiments to clear this up," says Jane. "That sounds like a great idea," says Dick tiredly.

Complete Foldover Design

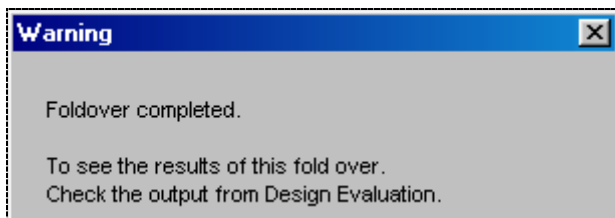
To untangle the main effects from the interactions we can fold over the design. Let's change all of the signs on the original eight runs and run this new fraction. The combined data will produce a Resolution IV design - all of the main effects will be free and clear of two-factor interactions, but all of the two-factor interactions will still be confounded with each other.

To create the new runs for the foldover design, click on the **Design** node and then select **Design Tools, Augment Design**. That brings up the following dialog box. Select **Fold over**. Click **OK** to continue.



Foldover Choice on Augmentation Dialog Box

By default, Design-Expert selects all the factors. This represents a complete foldover. Factors can be selected or cleared with a right click on the factor, or by double-clicking on the factor. Click **OK** to continue. The program puts up the following warning.



Suggestion Box After Selecting Foldover

This is really just a suggestion. Click **OK** to see the augmented design layout. Select **View, Std Order**. Notice how each run in block two is the reverse of block one. Complete foldover of a Resolution III design results in a Resolution IV design. See Montgomery's *Design and Analysis of Experiments* for the details on why this happens. In this case the new confounding scheme for the complete 16 runs, showing only main effects and two-factor interactions, comes out as shown on the following table.

Aliasing Before Foldover	Aliasing After Foldover
A = BD = CE = FG	A BD = CE = FG
B = AD = CF = EG	B AD = CF = EG
C = AE = BF = DG	C AE = BF = DG
D = AB = CG = EF	D AB = CG = EF
E = AC = BG = DF	E AC = BG = DF
F = AG = BC = DE	F AG = BC = DE
G = AF = CD = BE	G AF = BE = CD

Confounding Pattern Before and After Foldover

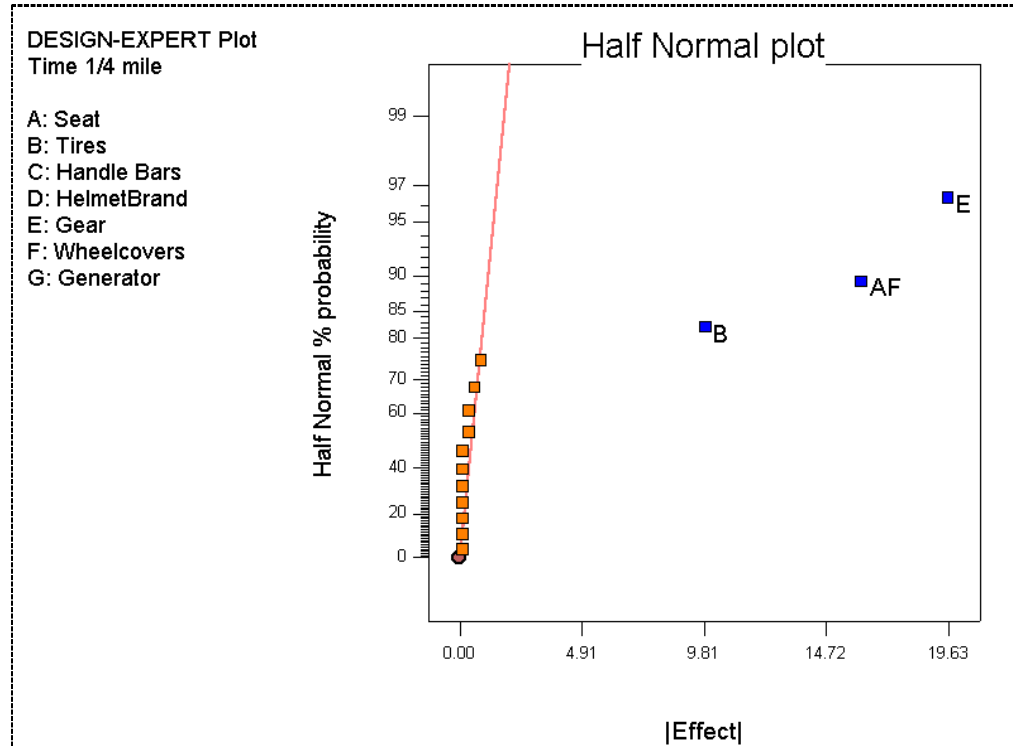
All main effects are now clear of two-factor interactions. Double-check this outcome, if you like, by doing a design evaluation. Dick's results for the additional runs can be seen in the following table.

Seat	Tires, Psi	Handle - bars	Helmet Brand	Gear	Wheel Covers	Generator	Time, Secs
Block 2							
Down	50	Down	Atlas	High	On	On	57
Up	50	Down	Windy	Low	On	Off	93
Down	40	Down	Windy	High	Off	Off	84
Up	40	Down	Atlas	Low	Off	On	87
Down	50	Up	Atlas	Low	Off	Off	94
Up	50	Up	Windy	High	Off	On	57
Down	40	Up	Windy	Low	On	On	86
Up	40	Up	Atlas	High	On	Off	83

Complete Foldover Design

To continue following along with Dick, you should now enter his times for the foldover block (Block 2) of eight runs. Be sure to change the design matrix to standard order first. The input factors must match up with the responses, otherwise the analysis will be nonsense.

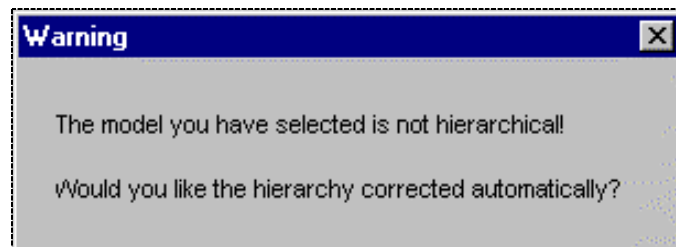
Click on **Time ¼ mile** and make the **Effects** plot. Starting from the right, pick the three largest effects.



Half-Normal Plot of Effects After Foldover

It now appears that what looked to be effect G really must be AF. Be careful though - recall that AF is aliased with CD and BE in this folded over design. From their subject matter knowledge, Dick and Jane know that AF, the interaction of the seat position with the wheel covers, cannot occur; and CD, the interaction of handle bar position with the brand of helmet, should not exist. They are sure the only feasible interaction is BE, the interaction of tire pressure and gears.

Proceed with the **ANOVA**. You will be warned that “**This design contains aliased terms.**” Click on **No** to continue. (You will go back later to the Alias List, so don’t worry.) The software gives the following warning.



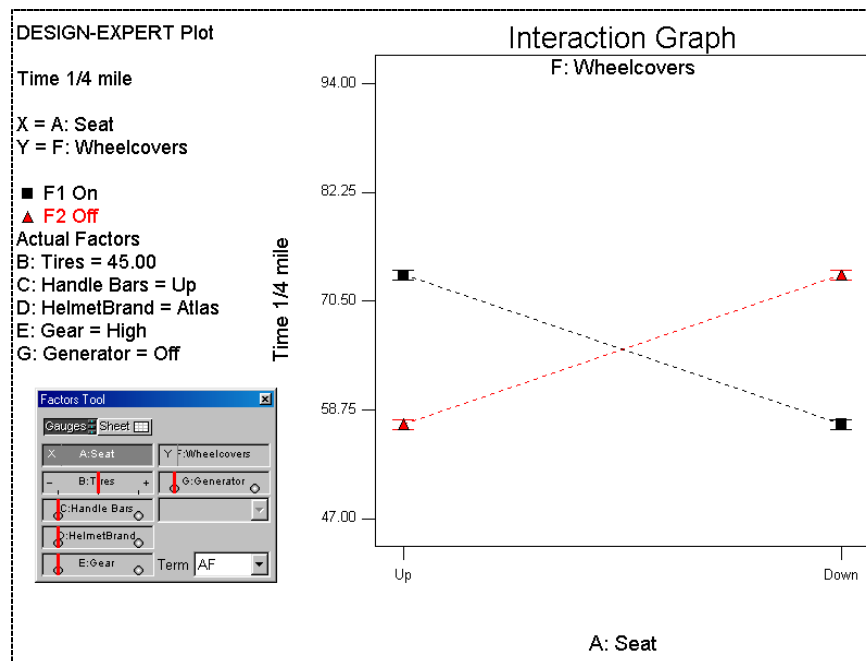
Warning that Model is not Hierarchical

Because you selected the interaction AF, the parent factors A and F must be included in the model to preserve hierarchy. This makes the model mathematically complete. Click **Yes** to correct the model for hierarchy. The ANOVA looks good. (Factor A is not significant but it's in the model to preserve hierarchy.)

ANOVA for Selected Factorial Model						
Analysis of variance table [Partial sum of squares]						
Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Block	410.06	1	410.06			
Model	2974.31	5	594.86	2089.27	< 0.0001	significant
A	0.56	1	0.56	1.98	0.1934	
B	390.06	1	390.06	1369.98	< 0.0001	
E	1540.56	1	1540.56	5410.76	< 0.0001	
F	3.06	1	3.06	10.76	0.0095	
AF	1040.06	1	1040.06	3652.90	< 0.0001	
Residual	2.56	9	0.28			
Cor Total	3386.94	15				

ANOVA Output After Foldover

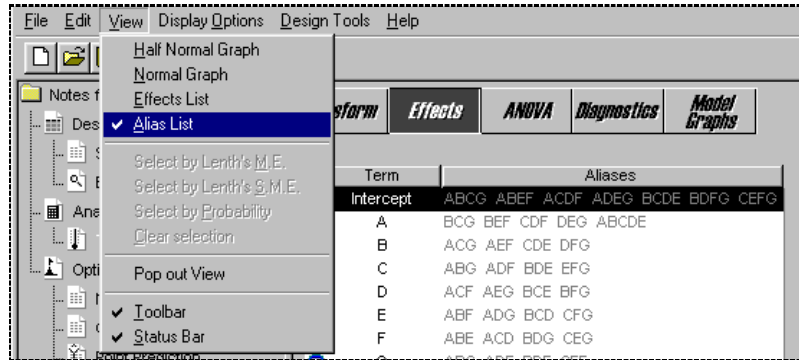
Click on **Diagnostics** and look at all the graphs. You will see nothing abnormal here. Then click on **Model Graphs** to view the “AF” interaction plot. Since neither “A” nor “F” as main effects change the response very much, the interaction forms an X.



The AF Interaction Plot

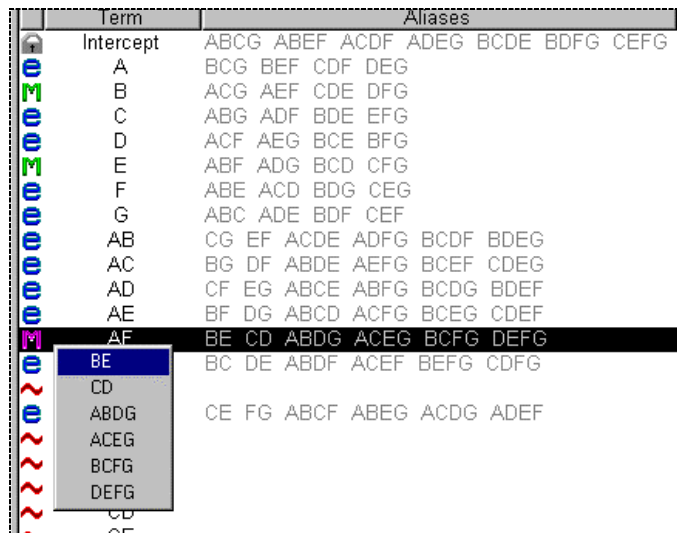
Investigation of Aliased Interactions

Dick decides to investigate the other interactions aliased with AF. You can follow along on your computer to see how this works. He clicks back on the **Effects** button and then on the **View** menu where he selects **Alias List**.



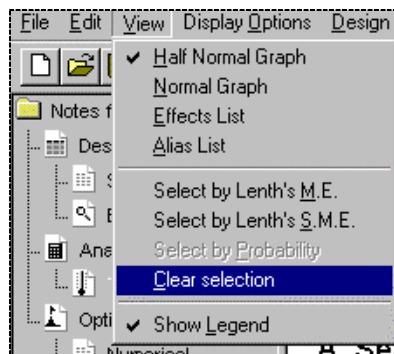
Viewing the Alias List

He locates and right clicks on the term **AF**, and selects **BE**.



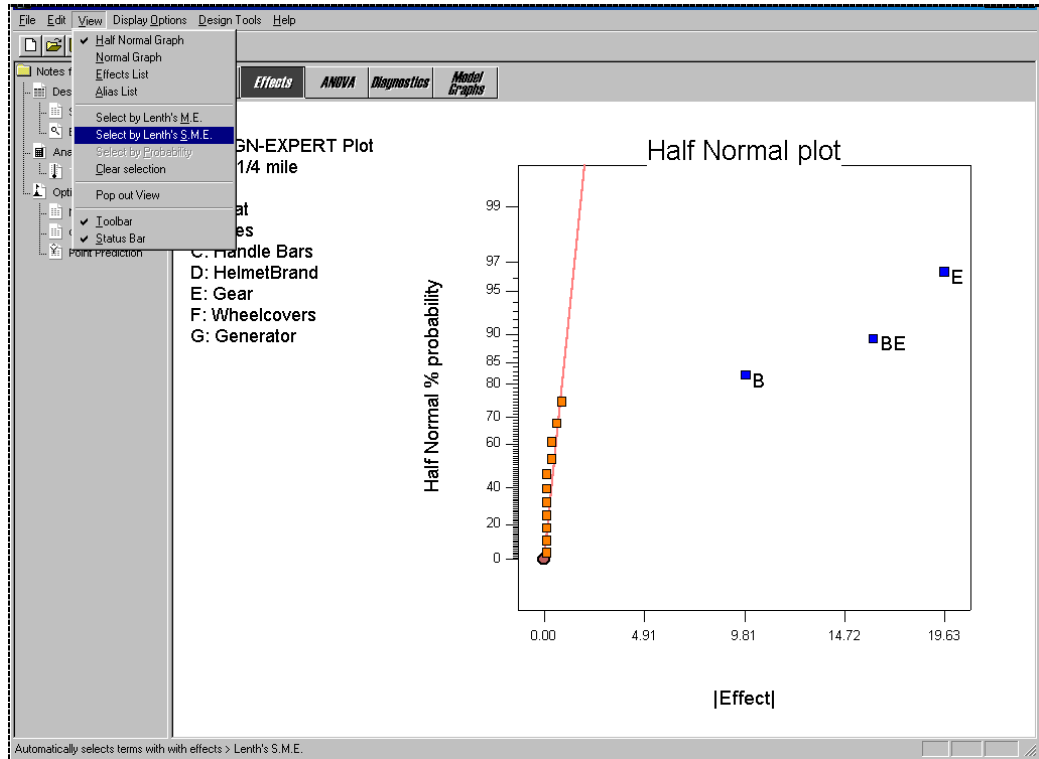
Substituting BE for AF

He then clicks the **View** menu where he re-selects **Half Normal Graph**. The effects of A and F remain from the prior analysis. They are not significant and no longer needed. Re-set the graph to its original state by going to **View, Clear selection**.



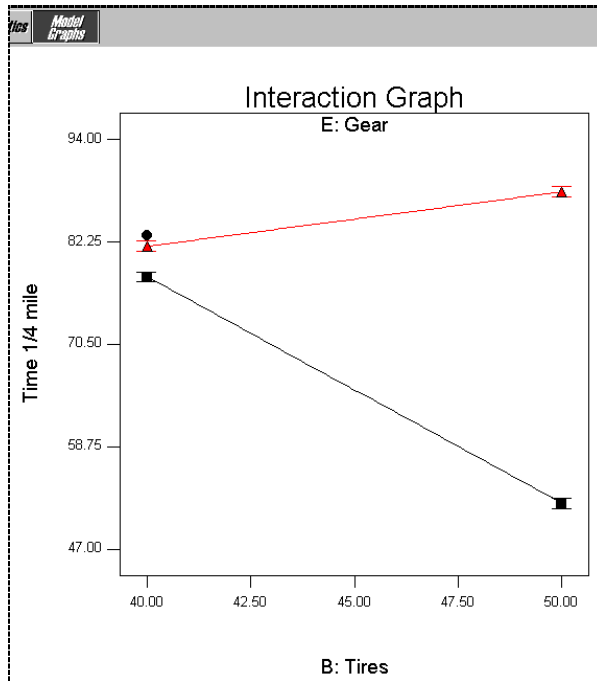
Clearing Effect Selections

Then let the program select the effects via **View, Select by Lenth's SME**.



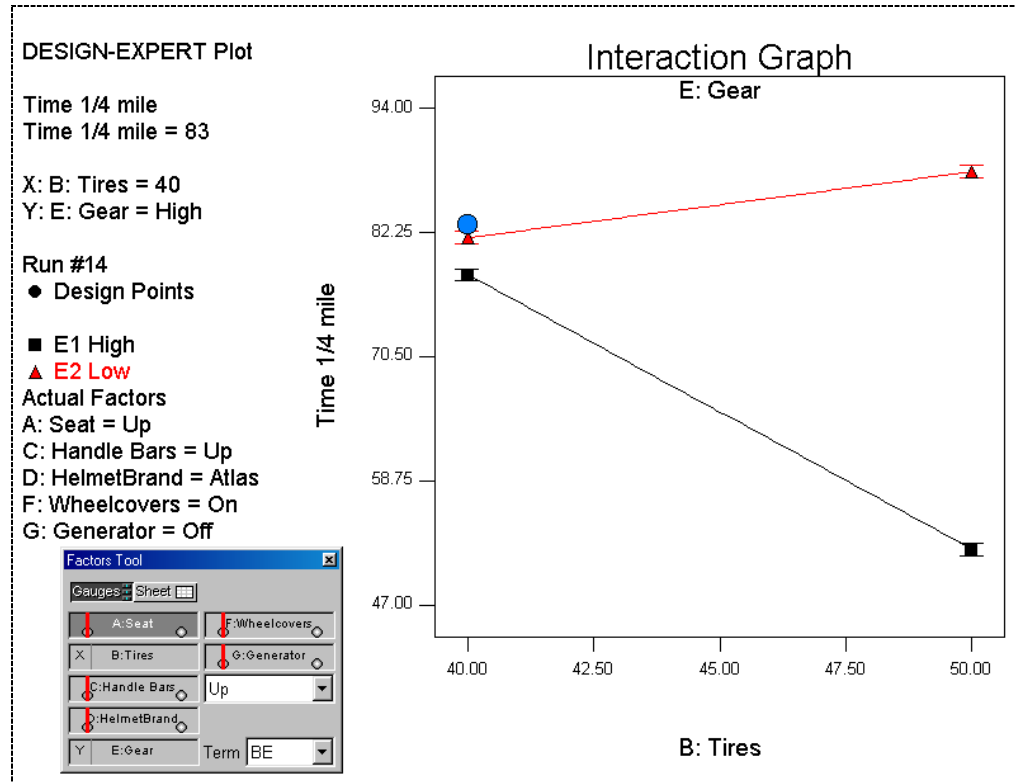
Selecting Effects on Half-Normal Graph After Substituting BE for AF on Master Model

Dick re-analyzes the data, selecting what's now labeled as BE and turning off A and F. He uses Design-Expert software to draw the interaction. Take a look at it yourself by clicking on the **Model Graphs** button. Dick feels that it really makes much better sense this way.



The BE Interaction Plot

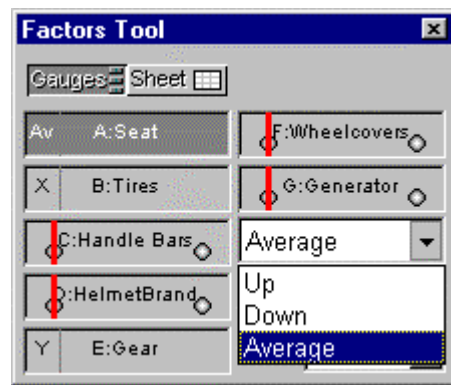
Notice that Design-Expert displays a point on the graph, which in fractional factorials, only pop up under certain conditions. Click on it to identify these conditions.



Identified Design Point (enlarged circle)

At the left of the graph you now see that at this point the tire pressure (B) equals 40 psi and the gear (E) is set high. Below that you see the run number identified (yours may differ due to randomization). The other factors, not displayed on the graph, would normally default to their average levels, but since they are categorical, the software arbitrarily chooses their low levels (red slide bars to the left). This setup matches the last row of the foldover, which produced a time of 83 seconds. (Note: due to randomization, your run number may differ from that shown to the left of the graph.)

If you prefer to display the results at an average of the low and high levels, click the list arrow and make this your selection as shown below.



Changing to an Average Level

Notice that the point on the graph now disappears because the average seat height was not one of the levels actually tested. At this point only factor A is set to the average level, but you can set other categorical factors at their average levels. If you like, set A back to the Up position. Then play with the other settings. See if you can find any other conditions at which an actual run was performed. Remember that even with the foldover, you've only run 16 out of the possible 128 (2^7) combinations, so you won't see very many points, if any, on the graphs.

Just to cover all bases, Dick decides to look at the second alias for AF – the CD effect, even though it just doesn't make sense that C (handlebars) and D (helmet brand) would interact. Dick sees that the interaction plots look quite different. What a surprise! If you can spare the time, check this out by going back to the Effects button and selecting to View, Alias List. Then replace BE with CD. (You must select the factors C and D in the model to make it hierarchical.) Based on his knowledge of biking, Dick makes a “leap of faith”: He assumes that BE is the real effect, not CD.

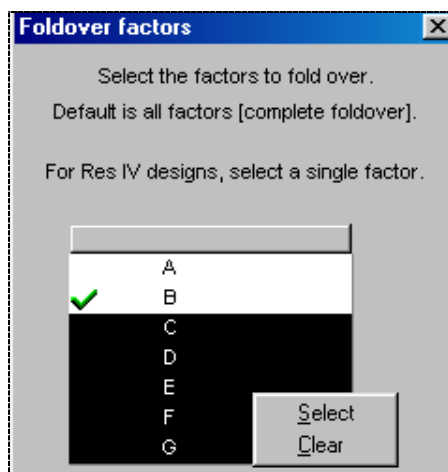
Now would be a good time to save your work by doing a File, SaveAs and typing in a new name, such as “Biker2”.

Single Factor Foldover Design

Let's say that after the first experiment Dick decided that the factors B and E were significant and suspected the interaction BE, not G, was also significant.

He discusses this with Jane. She declares, “By making just eight more runs you can prove your assumption.” “Wonderful,” sighs Dick, “Please show me how.” “Well,” replies Jane, “If you run the same points as in the first experiment, but reverse the pattern only for the B factor, then B and all of its two-factor interactions will be free and clear of any other two-factor interactions.” So Dick allows himself to be cajoled into another eight times around the track.

You can enter and analyze this data in your own computer by again doing a **File, Open Design** on the file **Biker.dx6** (do not save the current design). Then create the extra runs by selecting **Design Tools, Augment Design** and **Fold Over**. Double-click on factor **A** to deselect it. Then drag your mouse over factors **C** through **G**, then right-click and select **Clear**. Now only factor B remains selected for the foldover.



Single Factor Foldover on B

Press **OK** twice to see the augmented design. Then select **View, Std Order** to get it back in standard order. Note that the second half (Block 2) of this design is identical to the first half except for the inversion of the column for factor B (handlebars).

Enter the data from the following table of the results from Dick's second block of runs. Again, be certain everything matches properly.

Seat	Tires, Psi	Handle - bars	Helmet Brand	Gear	Wheel Covers	Generator	Time, Secs
Block 2							
Up	50	Up	Windy	Low	Off	Off	98
Down	50	Up	Atlas	High	Off	On	62
Up	40	Up	Atlas	Low	On	On	91
Down	40	Up	Windy	High	On	Off	88
Up	50	Down	Windy	High	On	On	61
Down	50	Down	Atlas	Low	On	Off	98
Up	40	Down	Atlas	High	Off	Off	89
Down	40	Down	Windy	Low	Off	On	92

Single Factor Foldover Design

We have computed by hand the confounding scheme, showing only main effects and two-factor interactions. You can evaluate this with the software via design

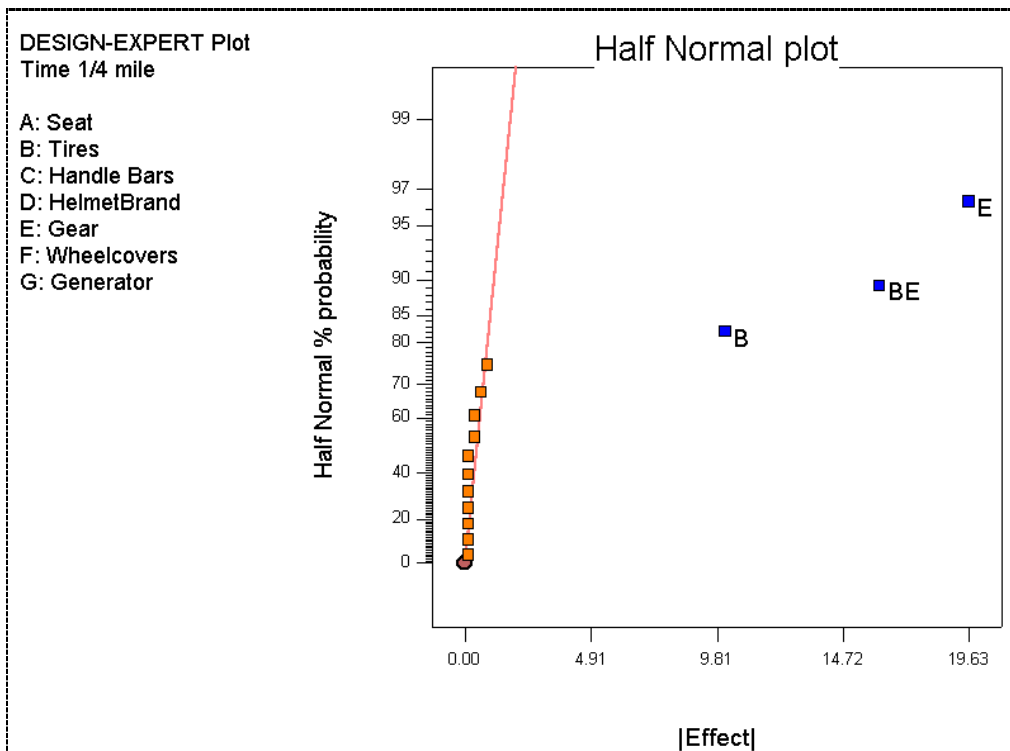
Evaluation, Results. Select for the **Order** the **2FI** model.

Aliasing Before Foldover	Aliasing After Foldover
A = BD = CE = FG	A = CE = FG BD
B = AD = CF = EG	AD = CF = EG B
C = AE = BF = DG	C = AE = DG BF
D = AB = CG = EF	D = CG = EF AB
E = AC = BG = DF	E = AC = DF BG
F = AG = BC = DE	F = AG = DE BC
G = AF = BE = CD	G = AF = CD BE

Confounding Pattern Before and After Single Factor Foldover

Notice that factor B and all of its two-factor interactions are now free of other main effects and two-factor interactions. When B changed sign in the second block of this design so did all of its two factor interactions. When the halves were combined, the B columns were no longer aliased with other effects, so they could be estimated independently.

Go ahead and analyze this new set of data. You will find B, E and the interaction BE to be significant.



Half-Normal Plot After Single Factor Foldover

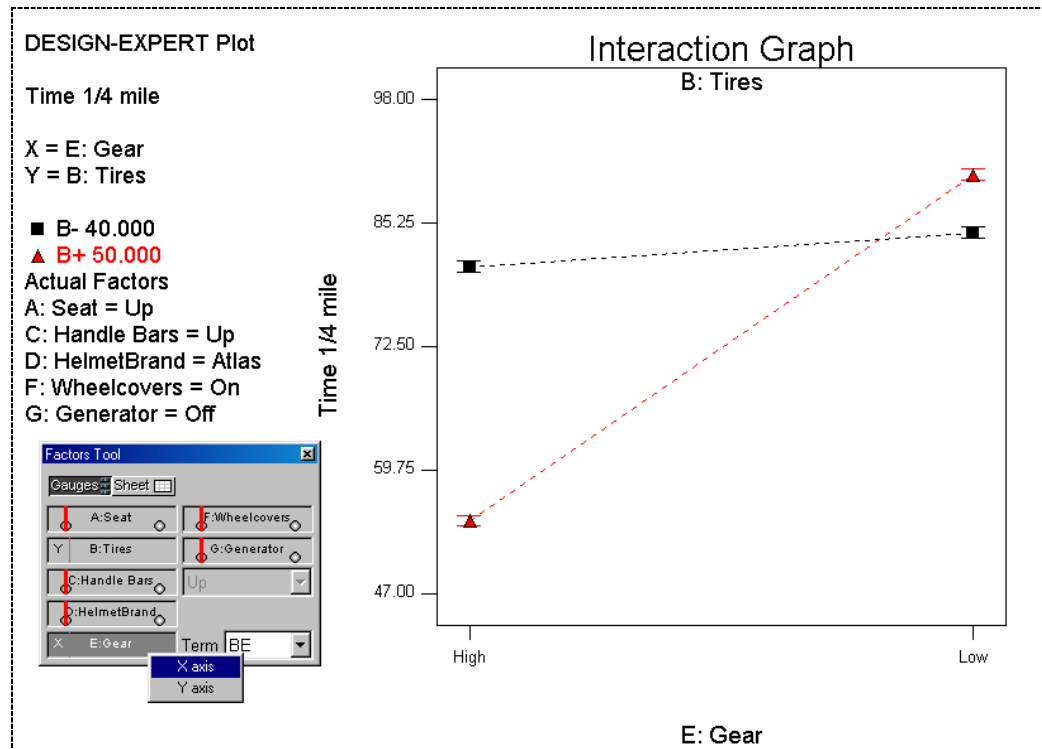
When you press **ANOVA**, the “**This design contains aliased terms**” warning again appears. Click on **Yes** to see the alias list.

Term	Aliases
Intercept	ACE AFG CDG DEF ACDF ADEG CEFG
A	CE FG CDF DEG ACDG ADEF
B	ABCE ABFG BCDG BDEF
C	AE DG ADF EFG ACFG CDEF
D	CG EF ACF AEG ACDE ADFG
E	AC DF ADG CFG ACFG CDEG
F	AG DE ACD CEG ACEF CDGF
G	AF CD ADE CEF ACEG DEFG
AB	BCE BFG BCDG BDEG
AC	
AD	CF EG ACG AEF CDE DFG
AE	
AF	
AG	
BC	ABE BDG ABDF BEFG
BD	BCG BEF ABCF ABEG ABCDE
BE	ABC BDF ABDG BCFG
BF	ABG BDE ABCD BCEG
BG	ABF BCD ABDE BCEF
CD	

Alias List Brought Up by Software

Click **ANOVA** and review the results. Then move on to **Diagnostics**. Check out all the graphs. Then click on **Model Graphs**. The interaction plot of BE looks the same as before. Check it out. Just to put a different perspective on things, right click on the **E: Gear** on **Factors Tool** and select it for the **X axis**. You now get a plot of EB, rather than BE. In other words, the axes are flipped. Note that the lines are now dotted

to signify that gear is a categorical factor (high or low). On the BE plot the lines were solid because B (tire pressure) is a numerical factor which can be adjusted to any level between the low and high extremes.



Interaction BE with Gear Chosen for X-Axis

Conclusion

“Thank goodness for designed experiments!” exclaimed Dick. “I know now that if I am in low gear then it doesn’t matter much what my tire pressure is, but if I am in high gear then I could improve my time with higher tire pressure.”

“Or,” cautioned Jane, “You could say that at low tire pressure it doesn’t really matter what gear you are in, but at high pressure you had better be in high gear!”

“Whatever,” moaned Dick.

“Another thing I noticed,” prodded Jane, “Your average time for each set of eight runs increased a lot. It was 70 seconds in the first experiment, 80 in the second set and almost 85 seconds in the last set of runs. You must have been getting tired. It’s a good thing we could extract the block effect or we might have been misled in our conclusions.”

“It doesn’t get any better than DOE,” agreed Dick.

In this instance, by careful augmentation of the saturated design through the foldover technique, Dick quickly found the significant factors among the seven he started with. He needed only 16 runs. A more conservative approach would be to start with a Resolution V design, where all main effects and two-factor interactions would be free from other main effects and two-factor interactions. But this would have required 64 experimental runs.

Generally the more conservative approach will be most appropriate. The lower resolution designs should be used only when absolutely necessary due to time constraints. In this case, Dick discovered a winning combination with the saturated Resolution III design. Then later, at his leisure, he determined via foldover exactly what caused the improvement.

