Section 6 – Response Surface Methods (RSM) Tutorials

Response Surface Design and Analysis

This tutorial shows the use of Design-Expert® software for response surface methodology (RSM). This class of designs is aimed at process optimization. A case study provides a real-life feel to the exercise.

Due to the specific nature of the case study, a number of features that could be helpful to you for RSM will not be exercised in this tutorial. Many of these features are used in the One Factor or Two-Level Factorial tutorials, so you will benefit by doing those first.

We will presume that you can handle the statistical aspects of RSM. You will find overviews in the on-line Help system. If you need in-depth education on RSM, we recommend you attend our Response Surface Methods For Process Optimization workshop. Call Stat-Ease or visit our website (www.statease.com) for a schedule.

The case study in this tutorial involves production of a chemical. The two most important responses, designated by the letter "Y", are:

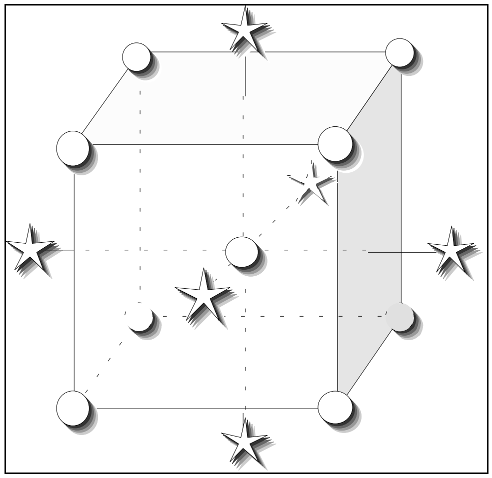
- Y_1 - Conversion (% of reactants converted to product)
- Y_2 - Activity.

The experimenter chose three process factors to study. Their names and levels can be seen in the following table.

Factor	Units	Low Level (-1)	High Level (+1)
A - time	minutes	40	50
B - temperature	degrees C	80	90
C - catalyst	percent	2	3

Factors for Response Surface Study

You will study the chemical process with a standard RSM design called a central composite design (CCD). It's well suited for fitting a quadratic surface, which usually works well for process optimization. The three-factor layout for the CCD is pictured below.



Central Composite Design for Three Factors

Assume that the experiments will be conducted over a two-day period, in two blocks:

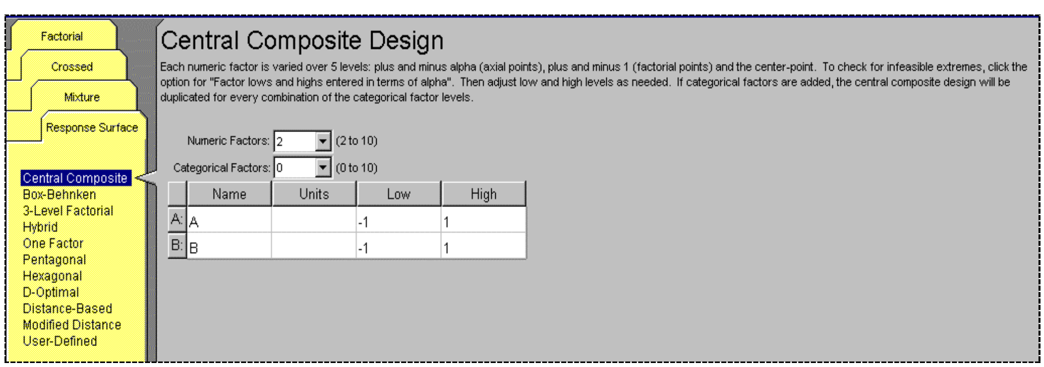
1. Twelve runs: composed of eight factorial points, plus four center points.
2. Eight runs: composed of six star (axial) points, plus two more center points.

Design the Experiment

Start the program by finding and double clicking the Design-Expert software icon. Take the quickest route to initiating a new design by clicking on the blank-sheet icon 🗋 on the left of the toolbar. The other route is via File, New Design (or associated Alt keys).

Main Menu and Tool Bar

Click on the **Response Surface** folder tab to show the designs available for RSM.



Response Surface Design Tab

The default selection is the Central Composite design, which will be used for this case study. The picture shows the layout of the CCD for two factors. Click on the choices for Box-Behnken and 3-Level Factorial to see what these alternative designs look like.

After exploring the Response Surface menu, re-select the **Central Composite** design. Then click on the entry field for **Numeric Factors** and enter **3**. (The maximum allowable number of factors is 10.) Ignore the option of including categorical factors in your designs (leave at default of 0).

Numeric Factors: 3 (2 to 10)

Categorical Factors: 0 (0 to 10)

	Name	Units	Low	High
A:	A		-1	1
B:	B		-1	1
C:	C		-1	1

Factor Selection

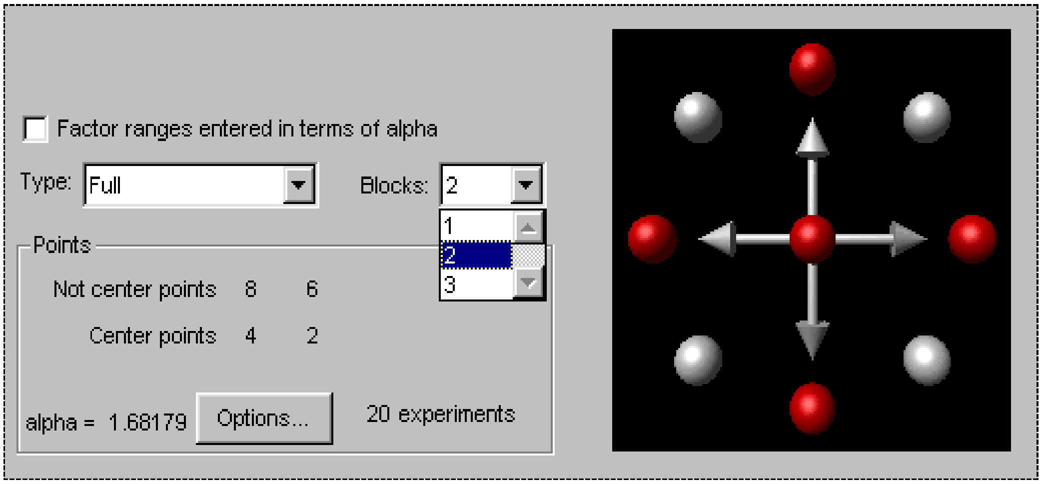
Replace the default entries for factor **Name** (A, B, C), and levels for **Low** (-1) and **High** (1), by tabbing or clicking to each cell and entering the details given in the introduction to this case study. The top of your screen should now match that shown below.

	Name	Units	Low	High
	time	min	40	50
	temperature	deg C	80	90
	catalyst	%	2	3

Completed Data Entry Form

The low and high values you entered above will be assigned to the limits of the factorial part of the design, i.e., the low and high levels will be coded -1 and +1. In the on-screen picture of the CCD, the factorial portion is represented by the gray points forming a box. The star (axial) points are located at the end of the arrows that come from the center point.

Now look at the bottom of the central composite design form. Leave the **Type** at its default value of **Full**. You will need two blocks for this design, one for each day, so click on the **Blocks** field and select **2**.

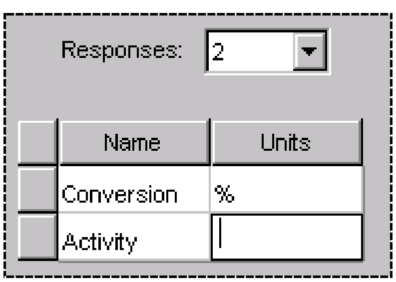


Central Composite Design Dialog Box - After Selecting 2 Blocks

The default selection of "Alpha," set at 1.68179 in coded units, is the axial distance from the center point and makes the design rotatable. A rotatable design provides equally good predictions at points equally distant from the center, a very desirable property for RSM. You can change alpha to other values via Options.

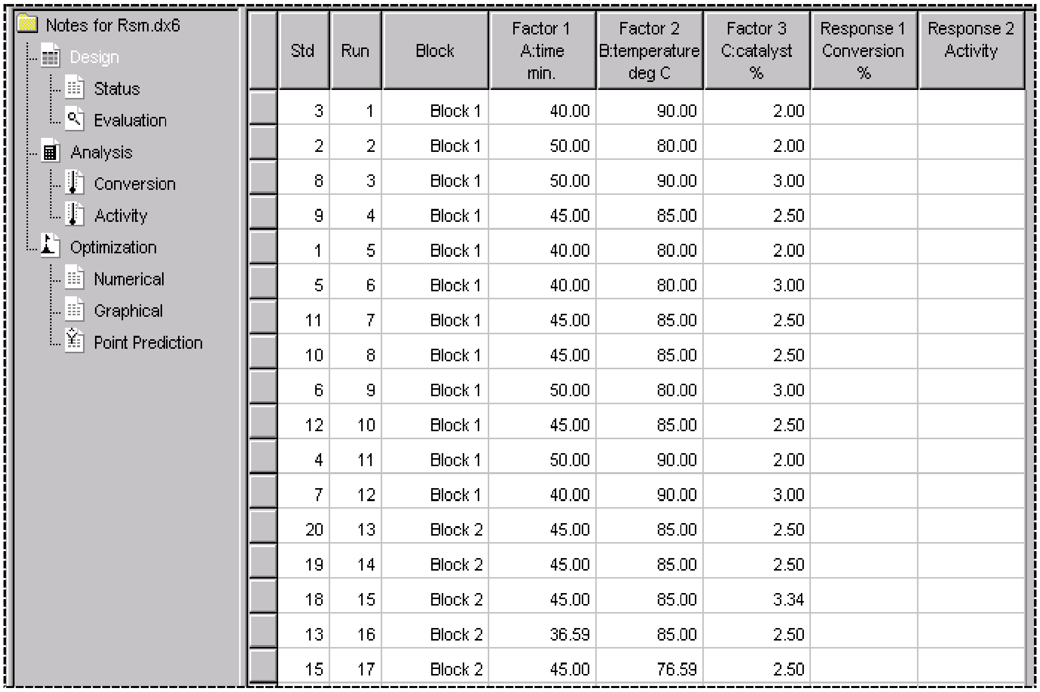
Normally the coded value for alpha exceeds the plus or minus one range of the factorial points, thus pushing the star points outside the factorial box. Therefore, if you enter low and high limits in terms of the factorial portion of the design, you must check the star points to be sure they don't exceed your operating limits. If you must keep your CCD within specified limits, you will need to check the "Factor ranges entered in terms of alpha" box and then enter the extreme values for each factor. The impact of doing this will be much clearer if you actually click the alpha box and see what happens, then click it again and leave it blank. Do this now. Notice that the ranges change to the more extreme values of the axial points.

Click on the **Continue** button to reach the second page of the "wizard" for building a response surface design. Select **2** from the pull down list for **Responses**. Then enter the response **Name** and **Units** for each response as shown below. At any time in the design-building phase, you can return to the previous page by pressing the Back button. Then you can revise your selections.



Completed Response Form

Press the **Continue** button to get the design layout.



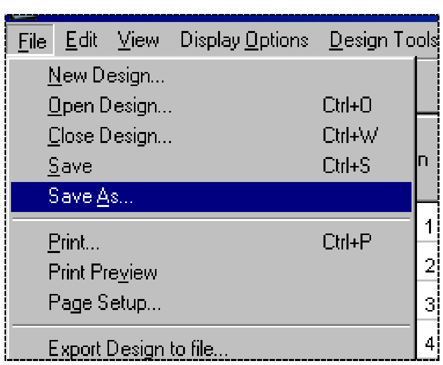
Std	Run	Block	Factor 1 A:time min.	Factor 2 B:temperature deg C	Factor 3 C:catalyst %	Response 1 Conversion %	Response 2 Activity
3	1	Block 1	40.00	90.00	2.00		
2	2	Block 1	50.00	80.00	2.00		
8	3	Block 1	50.00	90.00	3.00		
9	4	Block 1	45.00	85.00	2.50		
1	5	Block 1	40.00	80.00	2.00		
5	6	Block 1	40.00	80.00	3.00		
11	7	Block 1	45.00	85.00	2.50		
10	8	Block 1	45.00	85.00	2.50		
6	9	Block 1	50.00	80.00	3.00		
12	10	Block 1	45.00	85.00	2.50		
4	11	Block 1	50.00	90.00	2.00		
7	12	Block 1	40.00	90.00	3.00		
20	13	Block 2	45.00	85.00	2.50		
19	14	Block 2	45.00	85.00	2.50		
18	15	Block 2	45.00	85.00	3.34		
13	16	Block 2	36.59	85.00	2.50		
15	17	Block 2	45.00	76.59	2.50		

Completed Design Layout (Only partially shown, your run order may be different.)

The three columns on the left of the design layout identify the experimental runs. Design-Expert will randomize the order, so your runs will probably not match the layout shown above. If you like, adjust the widths of the columns by dragging them with your mouse wherever a double-ended arrow appears on your screen. By right-clicking at the top of a column, you are presented with an option to sort the design by any given variable. The right-click menu also offers an option called Edit Info, which allows you to reformat or change the factor levels. Try it! For example, right-click at the top of Factor 1 (A:time). Notice that you can also change factor names and units of measure. See the on-line Help system for more details on Edit Info and other ways in which you can manipulate the design layout.

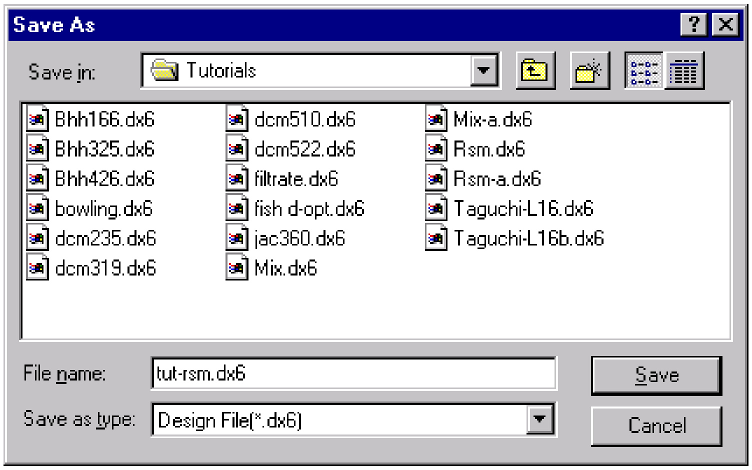
Save the Data to a File

Now that you've invested some time into your design, it would be prudent to save your work. Click on the **File** menu item and select **Save As**.



Save As Selection

You will be shown a standard dialog box that you can use to specify the name and destination of your data file.

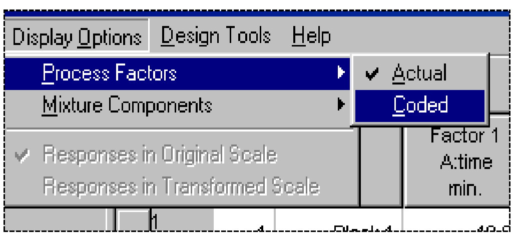


File Save As Dialog Box

Now you can enter a file name in the field with the default extension of **dx6**. (We suggest **tut-rsm.dx6**). Then, click on **Save**.

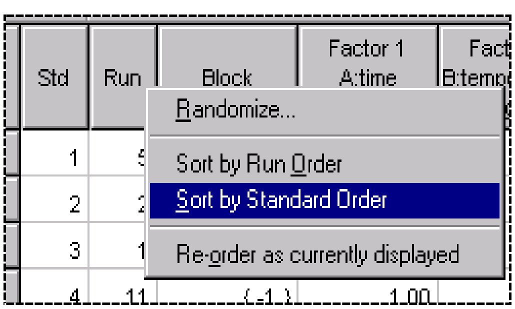
Examine the Design Runs

The design layout contains the completed design in run order showing the actual factor levels. You can also view this in coded factor levels by selecting **Display Options, Process Factors, Coded** from the menu bar.



Display Options menu for Coded Factor Levels

You can then right click on the **Run** column header and select **Sort by Standard Order** (or do the same on the Std column, or select View, Std Order from the menu).



Right Click Menu for Run Column

Std	Run	Block	Factor 1 A:time min.	Factor 2 B:temperature deg C	Factor 3 C:catalyst %	Response 1 Conversion %	Response 2 Activity
1	1	{ 1 }	-1.000	-1.000	-1.000		
2	12	{ 1 }	1.000	-1.000	-1.000		
3	7	{ 1 }	-1.000	1.000	-1.000		
4	5	{ 1 }	1.000	1.000	-1.000		
5	11	{ 1 }	-1.000	-1.000	1.000		
6	9	{ 1 }	1.000	-1.000	1.000		
7	2	{ 1 }	-1.000	1.000	1.000		
8	8	{ 1 }	1.000	1.000	1.000		
9	10	{ 1 }	0.000	0.000	0.000		
10	3	{ 1 }	0.000	0.000	0.000		
11	6	{ 1 }	0.000	0.000	0.000		
12	4	{ 1 }	0.000	0.000	0.000		
13	14	{ -1 }	-1.682	0.000	0.000		
14	13	{ -1 }	1.682	0.000	0.000		
15	20	{ -1 }	0.000	-1.682	0.000		
16	18	{ -1 }	0.000	1.682	0.000		
17	16	{ -1 }	0.000	0.000	-1.682		
18	17	{ -1 }	0.000	0.000	1.682		
19	19	{ -1 }	0.000	0.000	0.000		
20	15	{ -1 }	0.000	0.000	0.000		

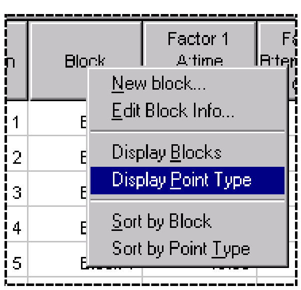
Design Layout with Coded Factor Levels in Standard Order

Add, delete or duplicate any experimental points by right clicking on the button (empty square) to the left of the row. (It's best to have your design in standard order before you do this.) If you add or delete runs, the experiments can be re-randomized by right clicking on the run column heading. You can change block names too, if you like. Go ahead and try some of these editing functions. See the Help system for more details.

Analyze the Results

Assume that you have completed the experiment. You now need to enter the responses into Design-Expert. We see no benefit to making you type all the numbers, particularly with the potential confusion due to differences in randomized run orders. Use the **File, Open Design** menu and select **Rsm.dx6** from the Design-Expert software program **Data** directory. Click on **Open** to load the data.

Let's examine the data. Move your cursor to the top of the **Block** column and do a right click. Choose **Display Point Type**.



Displaying the Point Type

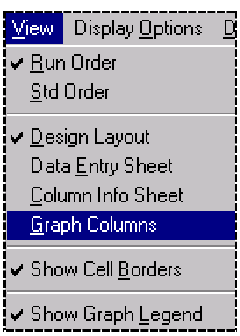
Again move your cursor to the top of the **Type** column (previously the Block column) and do a right click. This time select **Sort by Point Type**. The results can be seen below.

	Std	Run	Type	Factor 1 A:time min.	Factor 2 B:temperature deg C	Factor 3 C:catalyst %	Response 1 Conversion %	Response 2 Activity
	10	3	Center	45.00	85.00	2.50	75.0	60.4
	12	4	Center	45.00	85.00	2.50	83.0	60.6
	11	6	Center	45.00	85.00	2.50	76.0	59.1
	9	10	Center	45.00	85.00	2.50	81.0	59.2
	20	15	Center	45.00	85.00	2.50	91.0	58.9
	19	19	Center	45.00	85.00	2.50	80.0	60.8
	14	13	Axial	53.41	85.00	2.50	79.0	65.9
	13	14	Axial	36.59	85.00	2.50	76.0	53.6
	17	16	Axial	45.00	85.00	1.66	55.0	57.4
	18	17	Axial	45.00	85.00	3.34	81.0	63.2
	16	18	Axial	45.00	93.41	2.50	97.0	60.7
	15	20	Axial	45.00	76.59	2.50	85.0	60.0
	1	1	Fact	40.00	80.00	2.00	74.0	53.2
	7	2	Fact	40.00	90.00	3.00	66.0	59.8
	4	5	Fact	50.00	90.00	2.00	70.0	62.6
	3	7	Fact	40.00	90.00	2.00	88.0	53.4
	8	8	Fact	50.00	90.00	3.00	97.0	67.8
	6	9	Fact	50.00	80.00	3.00	90.0	67.9
	5	11	Fact	40.00	80.00	3.00	71.0	57.3
	2	12	Fact	50.00	80.00	2.00	51.0	62.9

Design Sorted by Point Type

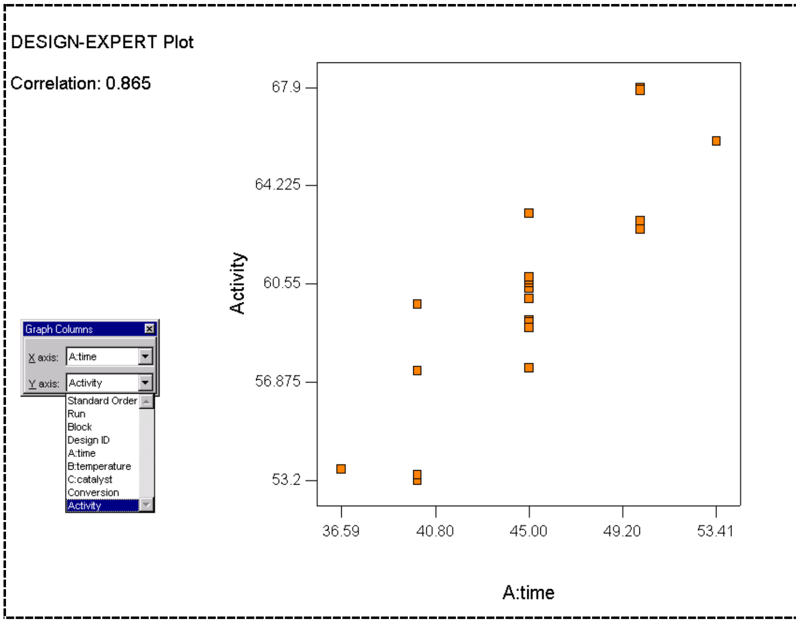
Now you can examine the results. Notice the variation in response for the replicated center points.

Before doing the analysis, it might be interesting to take a look at some simple plots. Select **View**, **Graph Columns**.



Graph Columns Feature for Design Layout

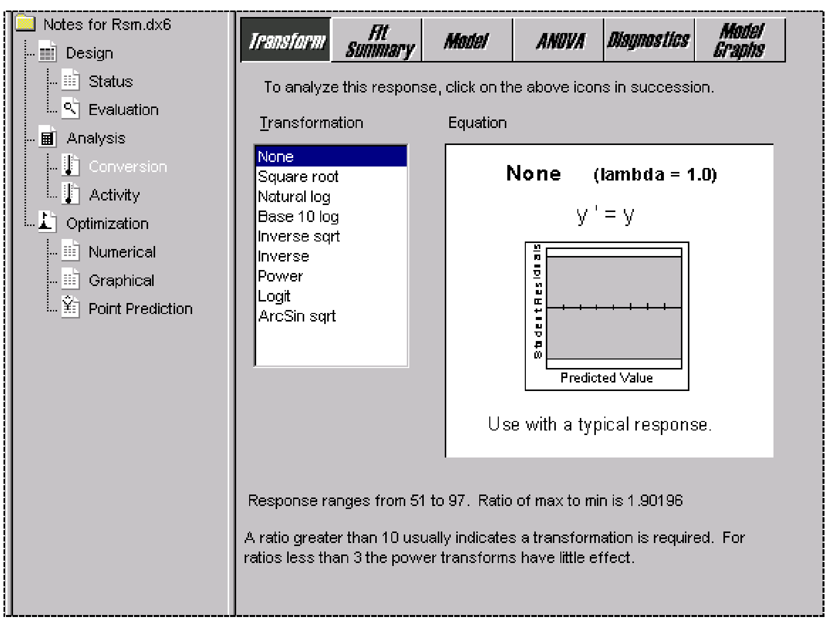
By default the graph shows a plot of factor A versus the first response. Click on the **Y-axis** and choose **Activity** (the second response). The graph shows a strong correlation between the levels of time and the activity response.



Graph of Factor A: Time versus Response: Activity

Go ahead and explore the data further by changing the x-axis and/or y-axis. You may want to graph individual factors versus one or the other response. In this way you can get a feel for the relationships. When you are done exploring, get back to the design layout screen by selecting **View**, **Design Layout**.

You will now start analyzing the responses numerically. Click on the node labeled **Conversion**. A new set of buttons appears at the top of your screen. They are arranged from left to right in the order needed to complete the analysis. What could be simpler?



Begin Analysis of Conversion

Design-Expert provides a full array of response transformations via the ***Transform*** option. See the on-line Help system for details. For now, accept the default transformation selection of None.

Click on the ***Fit Summary*** button next. At this point Design-Expert fits linear, two-factor interaction (2FI), quadratic and cubic polynomials to the response. The program displays a measure of progress during the calculations. For large data sets you may have to wait a few moments while your PC works. In most cases it will be done before you know it.

After the computations are complete, the program displays the results. To move around the display, use the side and/or bottom scroll bars. You will first see the identification of the response, immediately followed in this case by a warning: "The Cubic Model is Aliased." Do not be alarmed. By design, the central composite matrix provides too few unique design points to determine all of the terms in the cubic model. It's set up only for the quadratic model (or some subset). Next you will see several extremely useful summary tables for model selection. Each of these tables will be discussed briefly below.

The "Sequential Model Sum of Squares" summary table shows how terms of increasing complexity contribute to the total model. The model hierarchy is described below:

- "Linear": the significance of adding the linear terms to the mean and blocks.
- "2FI": the significance of adding the two factor interaction terms to the mean, block and linear terms already in the model.
- "Quadratic": the significance of adding the quadratic (squared) terms to the mean, block, linear and two factor interaction terms already in the model.
- "Cubic": the significance of the cubic terms beyond all other terms.

Transform | Fit Summary | Model | ANOVA | Diagnostics | Model Graphs

Response: Conversion

***** WARNING: The Cubic Model is Aliased! *****

Sequential Model Sum of Squares

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Mean	1.226E+005	1	1.226E+005			
Block	64.53	1	64.53			
Linear	763.05	3	254.35	1.96	0.1640	
2FI	1191.38	3	397.13	6.28	0.0083	
Quadratic	607.39	3	202.46	12.00	0.0017	Suggested
Cubic	30.95	4	7.74	0.32	0.8538	Aliased
Residual	120.90	5	24.18			
Total	1.254E+005	20	6269.80			

"*Sequential Model Sum of Squares*": Select the highest order polynomial where the additional terms are significant.

Summary Table: Sequential Model Sum of Squares

For each source of terms (linear, etc.), examine the probability ("PROB > F") to see if it falls below 0.05 (or whatever statistical significance level you choose). So far, the quadratic model looks best – these terms are significant, but adding the cubic order terms will not significantly improve the fit. (Even if they were significant, the cubic terms would be aliased, so they wouldn't be useful for modeling purposes.) Scroll down to the next table for lack of fit tests on the various model orders.

Lack of Fit Tests

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Linear	1845.37	11	167.76	6.38	0.0442	
2FI	653.99	8	81.75	3.11	0.1442	
Quadratic	46.60	5	9.32	0.35	0.8574	Suggested
Cubic	15.65	1	15.65	0.59	0.4836	Aliased
Pure Error	105.25	4	26.31			

"*Lack of Fit Tests*": Want the selected model to have insignificant lack-of-fit.

Summary Table: Lack of Fit Tests

The "Lack of Fit Tests" table compares the residual error to the "Pure Error" from replicated design points. If there is significant lack of fit, as shown by a low probability value ("Prob>F"), then be careful about using the model as a response predictor. In this case, the linear model definitely can be ruled out, because its Prob > F falls below 0.05. The quadratic model, identified earlier as the likely model, does not show significant lack of fit. Remember that the cubic model is aliased, so it should not be chosen.

Model Summary Statistics

Source	Std. Dev.	R-Squared	Adjusted R-Squared	Predicted R-Squared	PRESS	
Linear	11.40	0.2812	0.1374	-0.4682	3984.14	
2FI	7.95	0.7202	0.5803	0.3691	1711.98	
Quadratic	4.11	0.9440	0.8881	0.7891	572.20	Suggested
Cubic	4.92	0.9554	0.8396	-3.6399	12591.11	Aliased

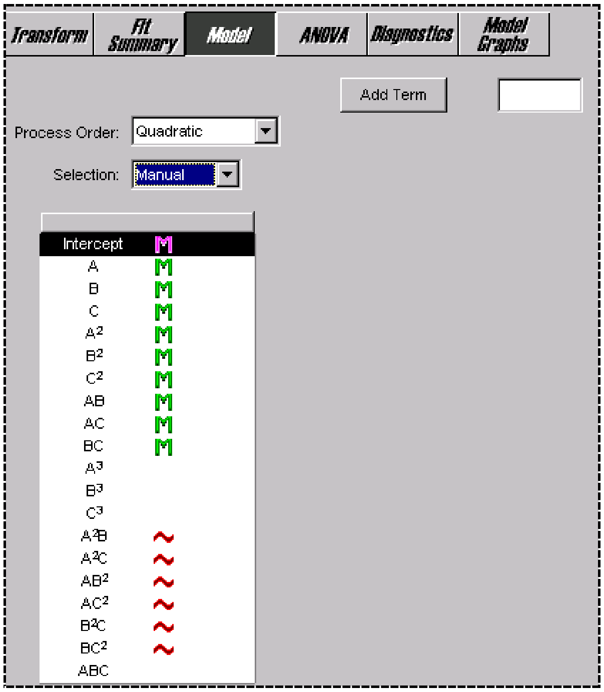
"*Model Summary Statistics*": Focus on the model maximizing the "Adjusted R-Squared" and the "Predicted R-Squared".

Summary Table: Model Summary Statistics

The "Model Summary Statistics" table lists other statistics useful in comparing models. The quadratic model comes out best: It exhibits low standard deviation ("Std. Dev."), high "R-Squared" values and a low "PRESS."

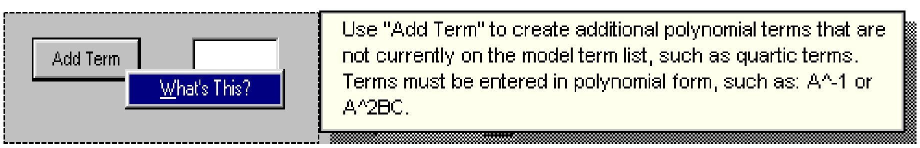
The program automatically underlines at least one "Suggested" model. Always confirm this suggestion by looking at these tables. See the on-line Help system for more information about the procedure for choosing model(s).

Design-Expert now allows you to select a model for an in-depth statistical study. Click on the ***Model*** button at the top of the screen next to see the terms in the model.



Model Results

At this stage you could press the Add Term button and insert higher degree terms with integer powers up to 6th order in total, up to a certain number of factors. Check Help for details. Also, remember that help is at your fingertips for program features. For example, if you place your mouse over the Add Term button and right-click you get the "What's This" option. Click it to get the following explanation.



Getting Help via the Right-Click "What's This?" Feature of Design-Expert

The program defaults to the "Suggested" model from the Fit Summary screen. If you want, you can choose an alternate model from the Process Order pull-down list. (Be sure to do this in the rare cases when Design-Expert suggests more than one model.) For this case study, we'll leave the selection at Quadratic. You could manually reduce the model by clicking off insignificant effects. For example, you will see in a moment that several terms in this case are marginally significant at best. Design-Expert also provides several automatic reduction algorithms as alternatives to the "Manual" method: "Backward," "Forward" and "Stepwise." Click the down arrow on the Selection list box to use these.

Click on the ***ANOVA*** button to produce the ANOVA table for the selected model. The ANOVA table is available in two views. The standard ANOVA will display all the statistical details necessary for analysis. If you choose View, Annotated ANOVA, you will get the same statistics with text providing brief explanations and guidelines. The software will default to whichever view was used last. The screen shown below is selected by choosing **View, ANOVA**.

Transform | Fit Summary | Model | ANOVA | Diagnostics | Model Graphs

Response: Conversion

ANOVA for Response Surface Quadratic Model

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Block	64.53	1	64.53		
Model	2561.82	9	284.65	16.87	0.0001
A	*14.44*	*1*	*14.44*	*0.86*	*0.3790*
B	*222.96*	*1*	*222.96*	*13.21*	*0.0054*
C	*525.64*	*1*	*525.64*	*31.15*	*0.0003*
A^2	*51.76*	*1*	*51.76*	*3.07*	*0.1138*
B^2	*119.19*	*1*	*119.19*	*7.06*	*0.0261*
C^2	*397.61*	*1*	*397.61*	*23.57*	*0.0009*
AB	*36.13*	*1*	*36.13*	*2.14*	*0.1774*
AC	*1035.13*	*1*	*1035.13*	*61.35*	*< 0.0001*
BC	*120.12*	*1*	*120.12*	*7.12*	*0.0257*
Residual	151.85	9	16.87		
Lack of Fit	*46.60*	*5*	*9.32*	*0.35*	*0.8574*
Pure Error	*105.25*	*4*	*26.31*		
Cor Total	2778.20	19			

Std. Dev.	4.11	R-Squared	0.9440
Mean	78.30	Adj R-Squared	0.8881
C.V.	5.25	Pred R-Squared	0.7891
PRESS	572.20	Adeq Precision	16.294

Statistics for Selected Model: ANOVA Table

The ANOVA in this case confirms the adequacy of the quadratic model (the Model Prob>F is less than 0.05.) You can also see probability values for each individual term in the model. You may want to consider removing terms with probability values greater than 0.10. Use process knowledge to guide your decisions.

Scroll down to bring the following table to your screen.

Factor	Coefficient Estimate	DF	Standard Error	95% CI Low	95% CI High	VIF
Intercept	81.60	1	1.69	77.77	85.43	
Block 1	-1.92	1				
Block 2	1.92					
A-time	1.03	1	1.11	-1.49	3.54	1.00
B-temperature	4.04	1	1.11	1.53	6.55	1.00
C-catalyst	6.20	1	1.11	3.69	8.72	1.00
A^2	-1.90	1	1.08	-4.34	0.55	1.02
B^2	2.88	1	1.08	0.43	5.33	1.02
C^2	-5.25	1	1.08	-7.70	-2.81	1.02
AB	2.13	1	1.45	-1.16	5.41	1.00
AC	11.38	1	1.45	8.09	14.66	1.00
BC	-3.87	1	1.45	-7.16	-0.59	1.00

Coefficients for the Quadratic Model

Following the ANOVA table you see the coefficients and associated confidence intervals for each term in the model. The mean effect shift for each block is listed here too.

Again scroll down to bring the next section to your screen.

Final Equation in Terms of Coded Factors:

Conversion =
+81.60
+1.03 * A
+4.04 * B
+6.20 * C
-1.90 * A^2
+2.88 * B^2
-5.25 * C^2
+2.13 * A * B
+11.38 * A * C
-3.87 * B * C

Final Equation for Conversion Response: Coded

Just below the model coefficients table you will find the model equation in terms of the coded factors and the model equation in the actual factors. Block terms are left out. These terms can be used to re-create the results of this experiment, but they cannot be used for modeling future responses.

Finally, by scrolling down a little further, you see the predicted values (which do include block effects), residuals and diagnostic information on the individual experiments. (You may need to drag the border between the icons on the left and the data table so you can get all the columns on screen. Give this a try.)

Diagnostics Case Statistics

Standard Order	Actual Value	Predicted Value	Residual	Leverage	Student Residual	Cook's Distance	Outlier t	Run Order
1	74.00	73.76	0.24	0.706	0.107	0.003	0.101	1
2	51.00	48.82	2.18	0.706	0.979	0.209	0.977	12
3	88.00	85.34	2.66	0.706	1.193	0.310	1.226	7
4	70.00	68.90	1.10	0.706	0.494	0.053	0.472	5
5	71.00	71.17	-0.17	0.706	-0.076	0.001	-0.071	11
6	90.00	91.73	-1.73	0.706	-0.774	0.131	-0.756	9

Residuals and Diagnostics (partially shown)

The note below the table ("Predicted values include block corrections.") alerts you that any shift from block 1 to block 2 will be included for purposes of residual diagnostics, but won't be seen in the equations shown above the table. The residuals (errors of prediction) should be carefully evaluated to ensure that you've satisfied the statistical assumptions underlying the analysis of variance. You will make plots of these diagnostics in a moment. The plots tell the story at a glance.

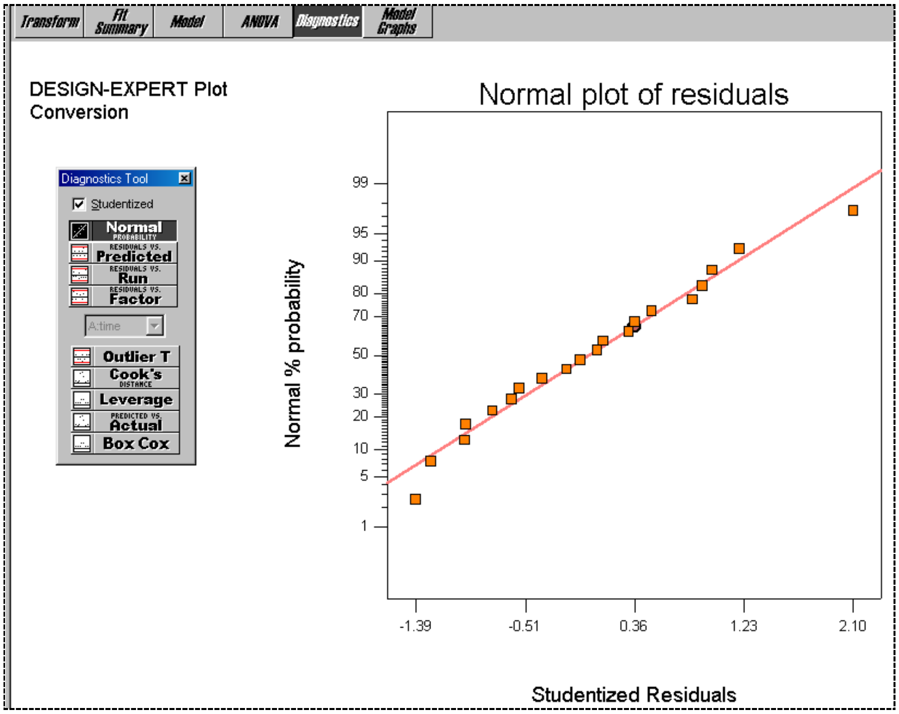
You cannot edit any of the ANOVA outputs. However, you can copy and paste the data to your favorite Windows word processor or spreadsheet.

Diagnose the Statistical Properties of the Model

The diagnostic details provided by Design-Expert can best be analyzed by inspection of various plots, which you will now generate. Click on the ***Diagnostics*** button. Residuals will be studentized unless you uncheck the first box on the floating tool palette (this is not advised). This counteracts varying leverages due to location of design points. For example, the centerpoints carry little weight in the fit and thus exhibit low leverage.

The most important diagnostic will be the normal probability plot of the studentized residuals. It comes up by default. You can drag the line with the mouse pointer to 'eye-fit' the residuals to a normal probability distribution. The data points should be approximately linear. A non-linear pattern (look for an S-shaped curve) indicates non-normality in the error term, which may be corrected by a transformation. There are no signs of any problems in our data. You can identify the source of any data point on the graph with a mouse click on the point.

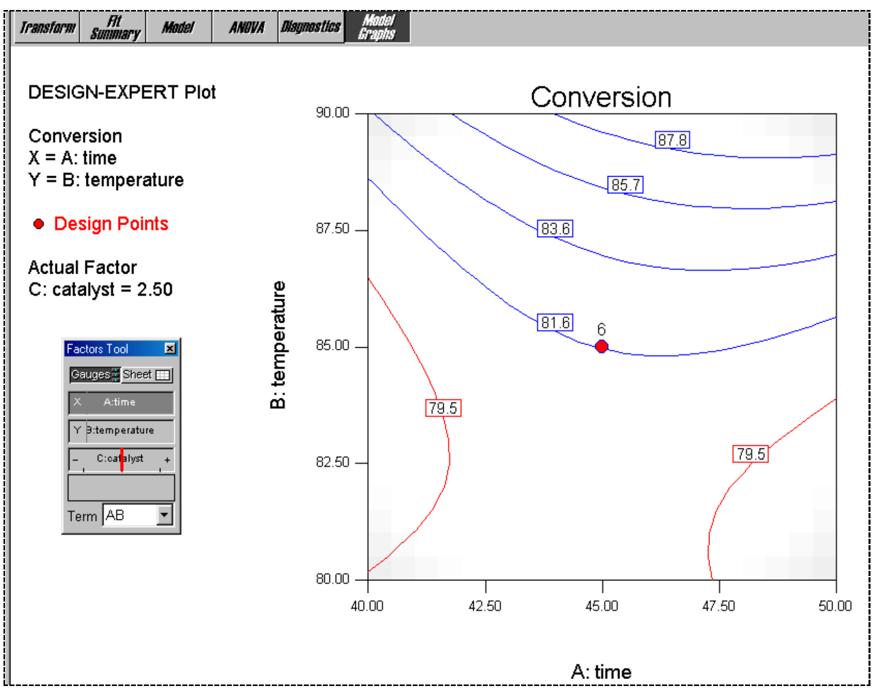
At the left of the screen you see the **Diagnostics Tool** palette. Each button on the palette represents a different diagnostics graph. Check out the other graphs if you like. Explanations for most of these graphs were covered in the Factorial Tutorials. In this case, none of the graphs indicate any cause for alarm.



Normal Probability Plot of the Residuals

Examine Model Graphs

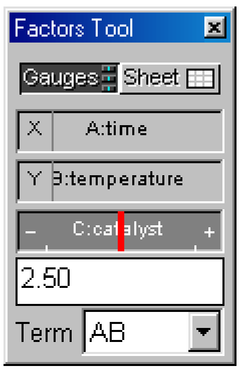
The diagnosis of residuals reveals no statistical problems, so you will now generate the response surface plots. Click on the ***Model Graphs*** button. The 2D contour plot of factors A versus B comes up by default.



Response Surface Plotting Range with Defaults

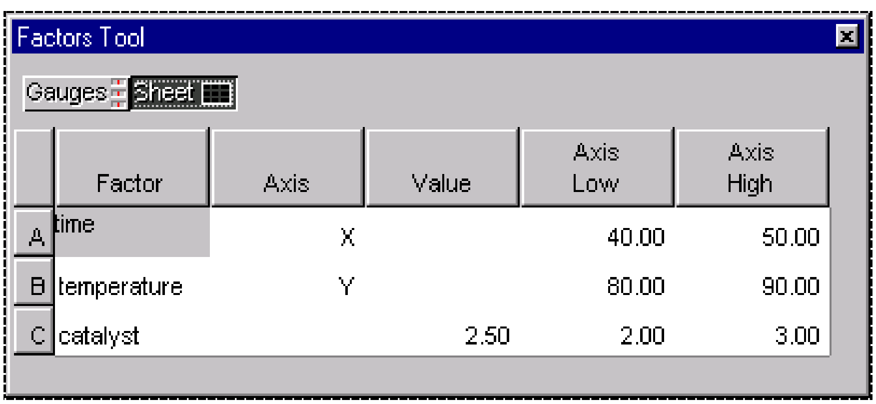
Note that Design-Expert will display any actual point included in the design space shown. In this case you see a plot of conversion as a function of time and temperature at a mid-level slice of catalyst. This slice includes six centerpoints as indicated by the dot at the middle of the contour plot. By replicating center points, you get a very good power of prediction at the middle of your experimental region.

The Factors Tool comes along with the default plot. Move this floating tool as needed by clicking on the top blue border and dragging it. The tool controls which factor(s) are plotted on the graph. The Gauges view is the default. Each factor listed will either have an axis label, indicating that it is currently shown on the graph, or a red slider bar, which allows you to choose specific settings for the factors that are not currently plotted. The red slider bars will default to the midpoint levels of the factors not currently assigned to axes. You can change a factor level by dragging the red slider bars or by right clicking on a factor name to make it active (it becomes highlighted) and then typing the desired level in the numeric space near the bottom of the tool palette. Click on the **C:catalyst** toolbar to see its value. Don't worry if it shifts a bit – we will instruct you on how to re-set it in a moment.



Factors Tool with Factor C Highlighted and Value Displayed

Switch to the Sheet View by clicking on the **Sheet** button.



Factors Tool Palette –Sheet View

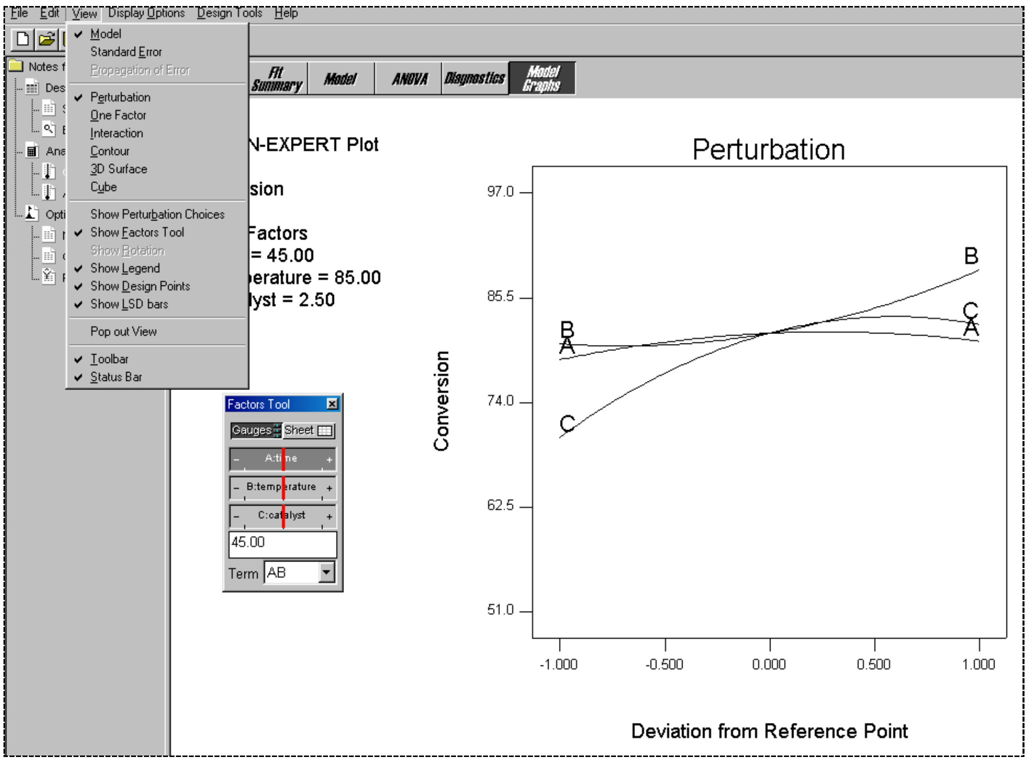
In the columns labeled Axis and Value you can change the axes settings or type in specific values for factors. If the catalyst value shifted from its mid-point value of 2.50, correct it now. Then return to the default view by clicking on the **Gauges** button.

At the bottom of the Factors Tool is a pull-down list from which you can also select the factors to plot. Only the terms that are in the model are included in this list. If you select a single factor (such as A) the graph will change to a One Factor Plot. If you choose a two-factor interaction term (such as AC) the plot will become the Interaction graph of that pair. The only way to get back to a Contour graph is to use the menu item View, Contour.

Perturbation Plot

Wouldn't it be handy to see all your factors on one response plot? You can do this with the perturbation plot, which provides silhouette views of the response surface. The real benefit from this plot is for selecting axes and constants in contour and 3D plots.

Use the **View, Perturbation** menu item to select it.



The Perturbation Plot

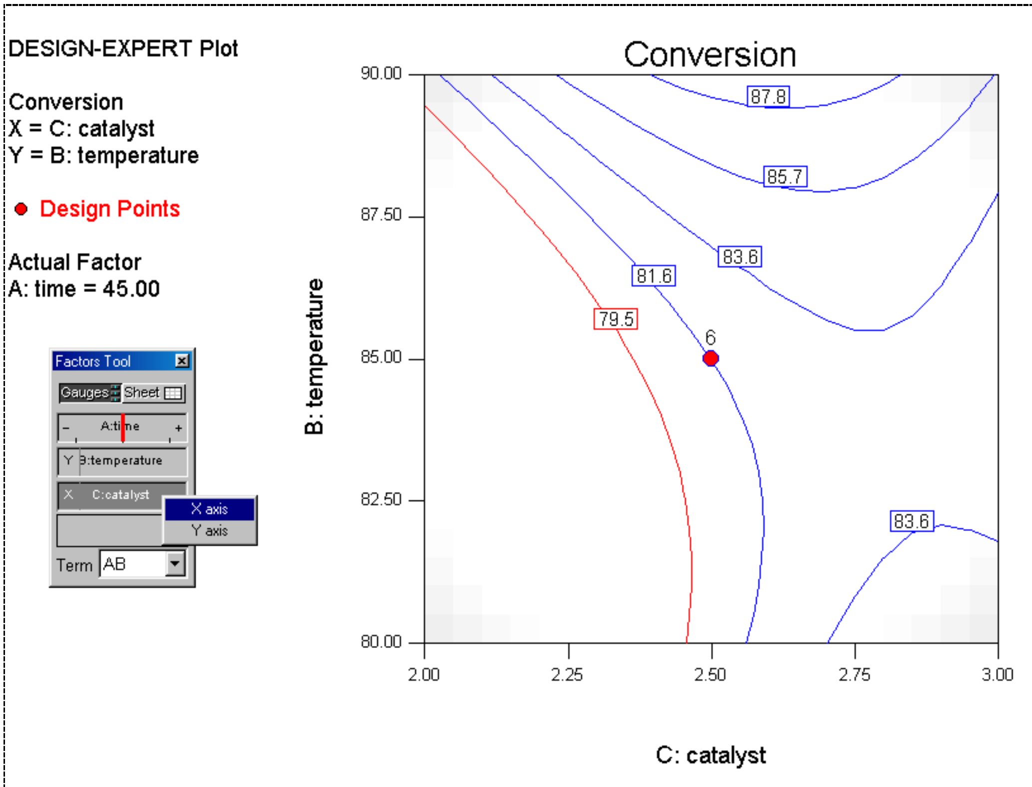
For response surface designs the perturbation plot shows how the response changes as each factor moves from the chosen reference point, with all other factors held constant at the reference value. Design-Expert sets the reference point default at the middle of the design space (the coded zero level of each factor).

Click on the curve for factor A to see it better. (The software will highlight it with a different color.) In this case, you can see that factor A (time) produces a relatively small effect as it changes from the reference point. Therefore, because you can only plot contours for two factors at a time, it makes sense to choose B and C, and slice on A.

Contour Plot

Let's look at the contour plot of factors B and C. Return to the contour plots via the **View**, **Contour** selection. Right click on the **catalyst** bar in the **Factors tool** palette. Then select **X axis** by left clicking on it.

You now see a catalyst versus temperature plot of conversion, with time held as a constant at its midpoint.



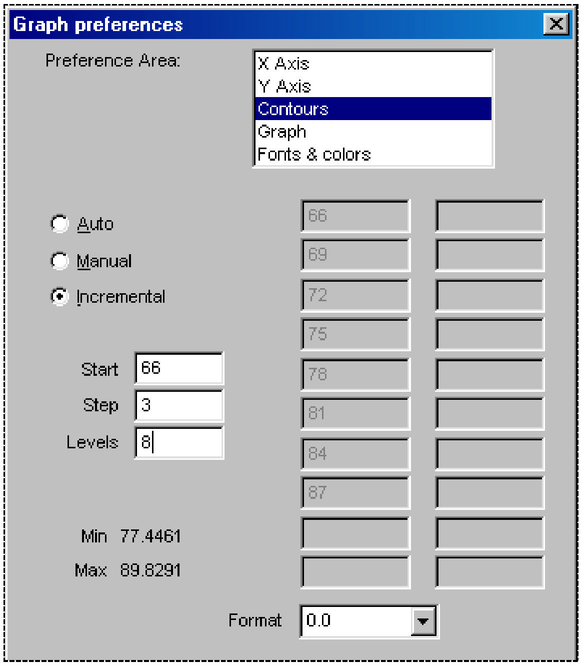
Response Surface - Time Constant

If you like, change the time constant by dragging the slider left and right. See how it affects the shape of the contours. Also, notice how the plot darkens as you approach the extremes. You're getting into areas of extrapolation. Be careful out there! When you're done, put the slider back at its centerpoint.

Design-Expert draws five contour levels by default. They range from the minimum response to the maximum response. Click on a contour to highlight it. You can move the contours by dragging them to new values. (Place the mouse cursor on the contour and hold down the left button while moving the mouse.) Give this a try – it's fun! Add new contours or set flags via a right mouse click. Check it out.

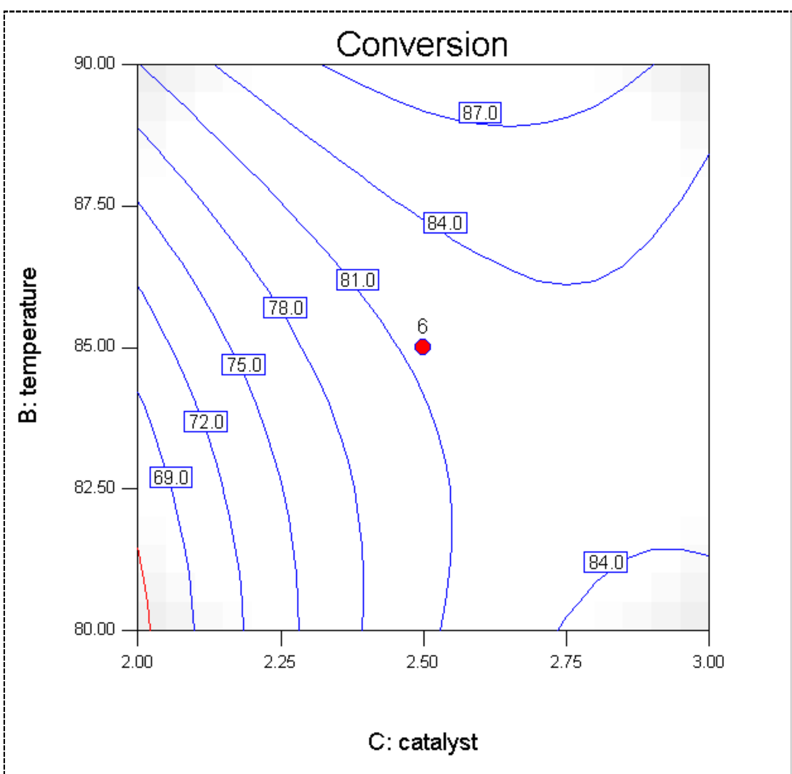
To get more precise contour levels for your final report, you could right-click each one and enter the desired value. Check this out if you like. But we recommend another approach: right click in the drawing or label area of the graph and choosing **Graph Preferences**. Then choose **Contours**.

Now select the **Incremental** option and fill in the **Start** at **66**, **Step** at **3** and **Levels** at **8**. Your screen should now match that shown below.



Contours Dialog Box: Incremental Option

Press **OK** to get a good-looking contour plot.

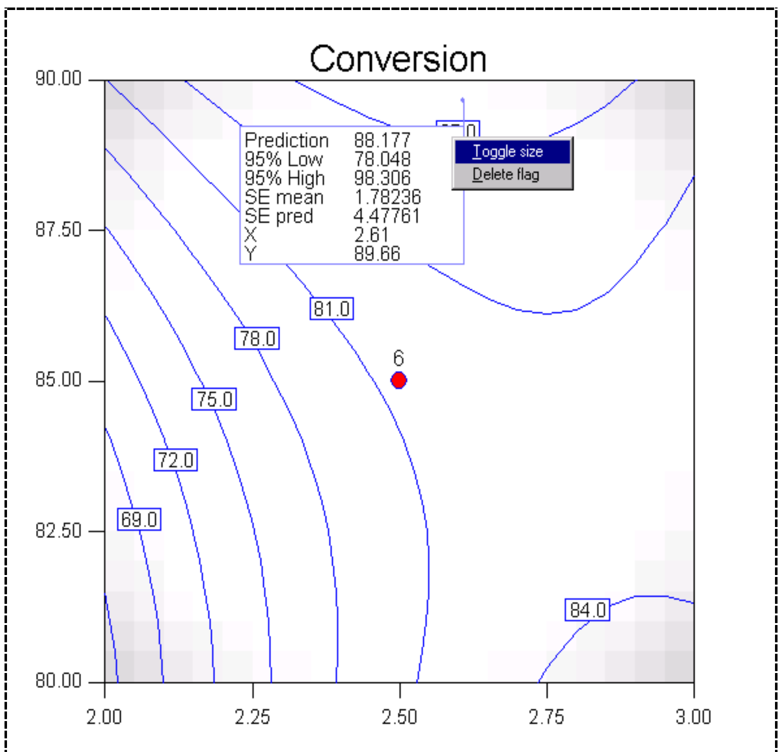


Contour Plot with New Contour Values

Right clicking on a contour gives you the following options: Delete contour, Set contour value, Add flag, Add contour and Graph Preferences. Check it out. (First left click on a contour line to highlight it, then right click to get the menu options.)

Options Available with a Right Click on a Contour Line

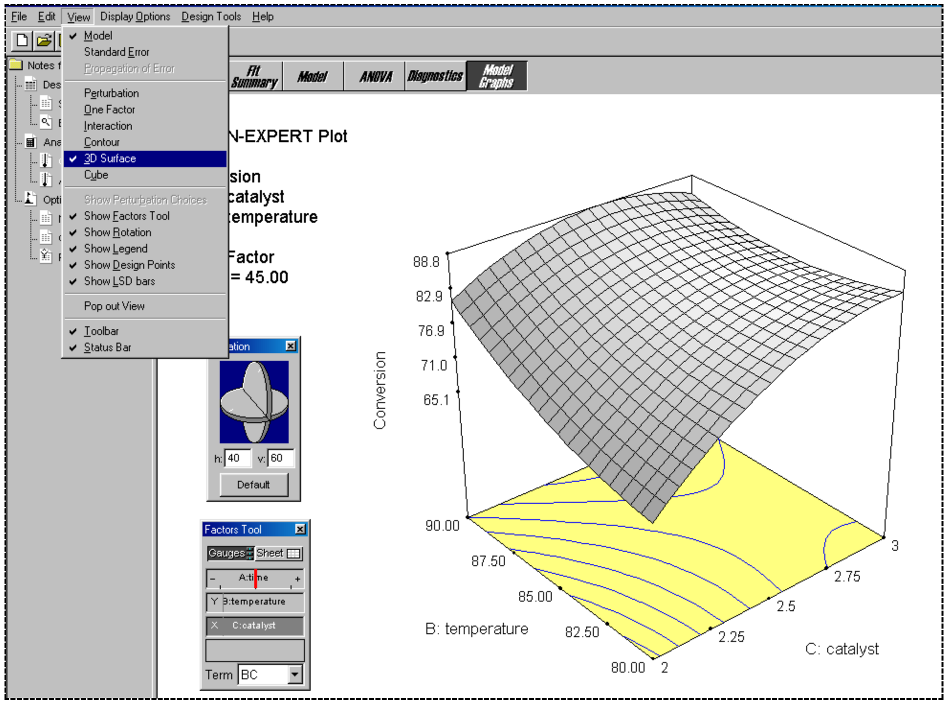
Move your mouse to a spot near the top of the graph, where the response hits the maximum. Click the right mouse button and **Add flag**. It displays the value of the response at that point. Now do a right click of the mouse on the flag and select **Toggle size**. The expanded flag displays a 95 percent confidence interval on the prediction. Use this to 'manage expectations' about individual confirmation runs near this point. Natural variability plus imprecision in the estimate of the mean (SE mean) will cause actual outcomes to differ from the prediction. The larger flag also lists the point coordinates.



Contour Plot with Larger Flag Planted

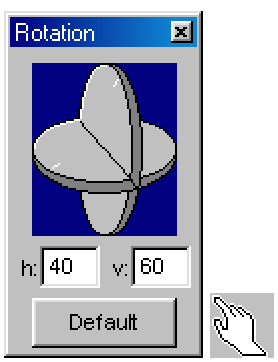
If you have a printer, you can print the contour plot by using the File, Print menu item.

Selecting **View, 3D Surface** will result in a three-dimensional display of the response surface. If you want nicer numbers on the response axis, do a right click to bring up the graph preferences and change values for the Z-axis. Check it out.



3D Response Surface Plot

The **Rotation** tool allows you to view the 3D surface plot from any angle.



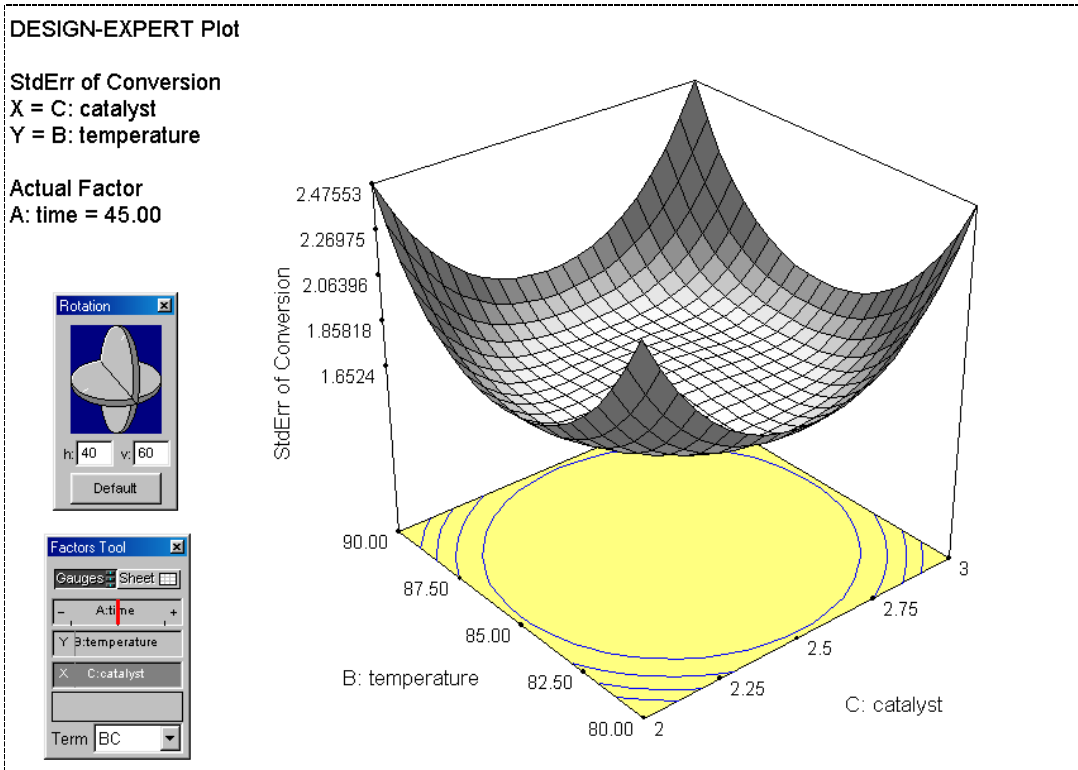
Control for Rotating 3D Response Plot

Move your cursor over the tool. The pointer changes to a hand. Now use the hand to rotate the vertical or horizontal wheel. Watch the 3D surface change. It's fun! Press the Default button when you're done playing. The graph then re-sets to its original position. Notice that you can also specify the horizontal ("h") and vertical ("v") coordinates.

Remember that you're only looking at a 'slice' of factor A (time) at its center point. Normally, you'd want to make additional plots with slices of A at the minus and plus one levels. But let's move on to another plot provided by Design-Expert software.

Standard Error Plot

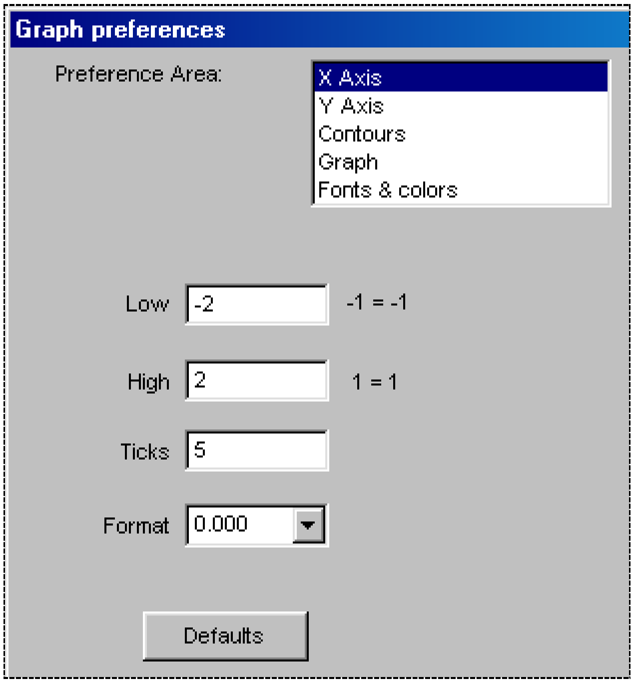
The standard error plot shows how the variance associated with prediction changes over your design space. The shape depends only on input factors, so you really should look at this plot before you gather data. It's a great tool for evaluating your design. To generate this plot, use the **View, Standard Error** menu selection.



3D Plot of Standard Error

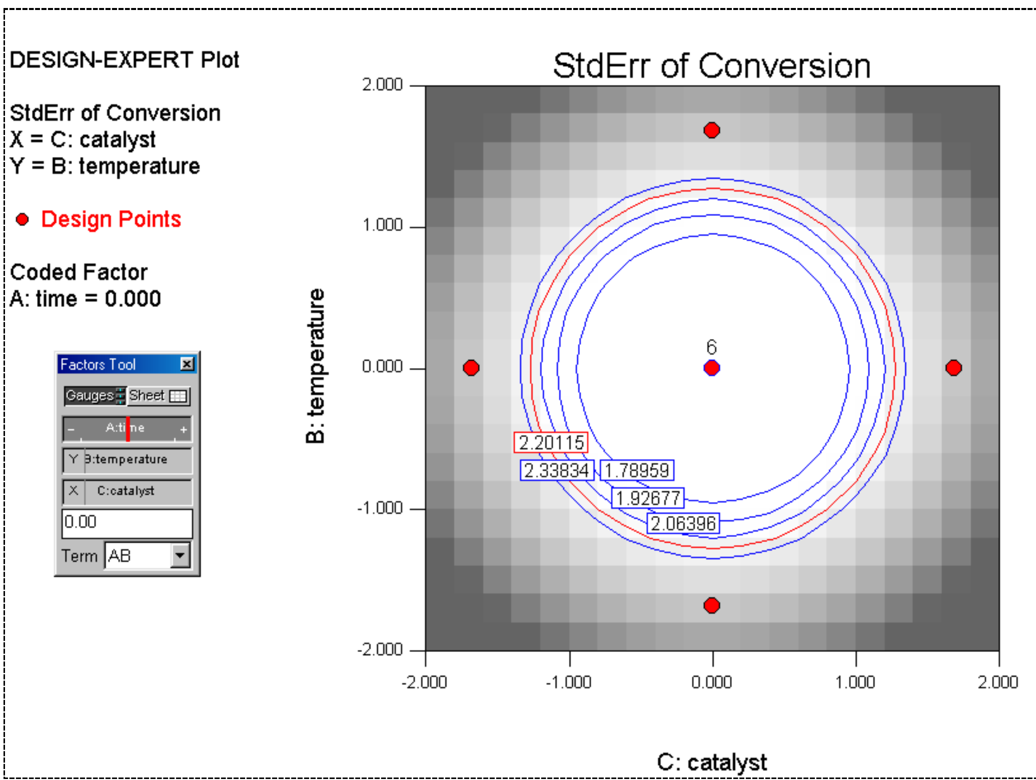
You can see that the central composite design provides relatively precise predictions over a broad area around the centerpoint. Also, notice the circular contours. This indicates the statistical property of rotatability, another desirable feature of central composite design.

You can manipulate the standard error plot just like the response plots. Choose **View, Contour** to get a better view of the circular contours. Let's see what happens if you extrapolate beyond the region of experimentation. First select **Display Options**, **Process Factors, Coded**. Then do a right mouse click on the graph and select **Graph preferences**. Change the default **X Axis** value for **Low** to **-2** and the **High** value to **2**.



Changing X-axis Values

Next, click on the **Y Axis** tab and change the **Low** value to **-2** and the **High** value to **2**. After making all these changes to the graph properties, press **OK**. (If you see any leftover flags, right click on them and delete.)

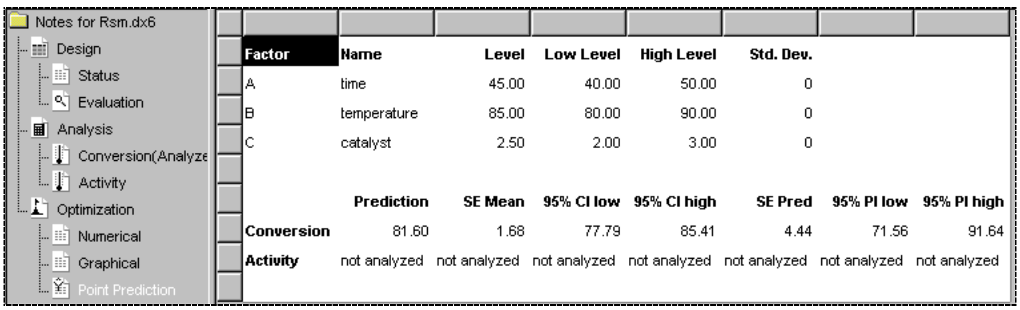


Contour Plot of Standard Error with Expanded Axes, Extrapolated Area Shaded

Notice that the corners of the contour plot are shaded. These areas cannot be predicted as well as the interior region. Design-Expert provides the shading as a warning against extrapolation. It begins at one-half the standard deviation and increases linearly up to 1.5 times the standard deviation. You will also see the shading on response (model) plots. So long as you stay within the specified factorial ranges, the pictures will remain relatively clear, but if you widen the ranges, don't be surprised to see considerable darkening. Be wary of predictions in these nether regions! Remember that the darker shading represents higher standard error. You now can see that it's dangerous to extrapolate outside the plus or minus one region, the factorial portion of this central composite design. Select **Display Options**, **Process Factors, Actual** to view the factors in their original units of measure.

Response Prediction

This feature in Design-Expert software allows you to generate predicted response(s) for any set of factors. To see how this works, click on the **Point Prediction** icon in the optimization section (lower left on your screen).



Factor	Name	Level	Low Level	High Level	Std. Dev.		
A	time	45.00	40.00	50.00	0		
B	temperature	85.00	80.00	90.00	0		
C	catalyst	2.50	2.00	3.00	0		
	Prediction	SE Mean	95% CI low	95% CI high	SE Pred	95% PI low	95% PI high
Conversion	81.60	1.68	77.79	85.41	4.44	71.56	91.64
Activity	not analyzed	not analyzed	not analyzed	not analyzed	not analyzed	not analyzed	not analyzed

Point Prediction Data Table

The Factors Tool again allows you to adjust the settings to any values you wish. Go ahead and play with them now if you like. You can either move the slider controls, or switch to the Sheet view and enter values.

Be sure to look at the 95% prediction interval ("PI low" to "PI high"). This tells you what to expect for an individual confirmation test. You might be surprised at the level of variability, but it will help you manage expectations. *(Note: block effects are not accounted for in the prediction.)* You can print the results by using the File, Print command.

Analyze the Data for the Second Response

This step is a BIG one. Analyze the data for the second response, activity. Be sure you find the appropriate polynomial to fit the data, examine the residuals and plot the response surface. *Hint: The correct model is linear.*

Before you quit, do a **File**, **Save** to preserve your analysis. Design-Expert will save your models. To leave Design-Expert, use the File, Exit menu selection. The program will warn you to save again if you've modified any files.

Response Surface Optimization Tutorial

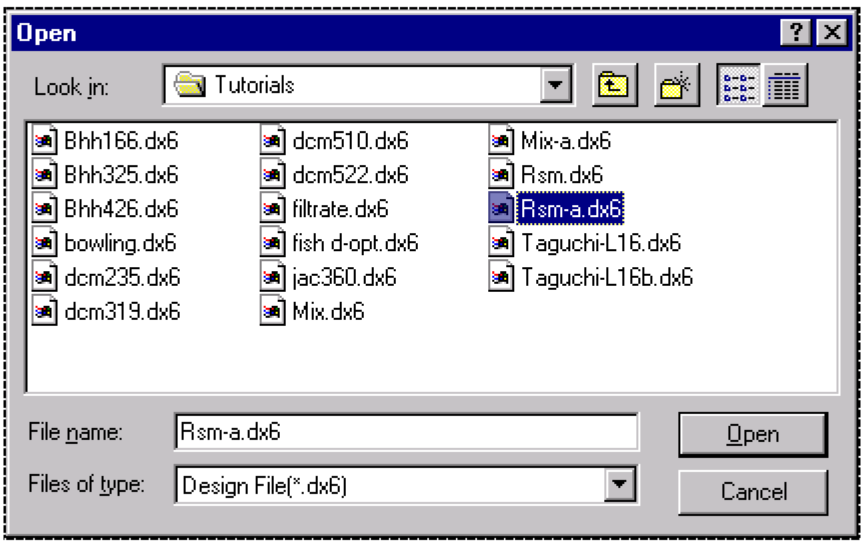
This tutorial shows the use of Design-Expert software for optimization experiments. It's based on the data from the Response Surface Tutorial. You should go back to this section if you've not already completed it.

For details on optimization, see the on-line program help. Also, Stat-Ease provides in-depth training in its Response Surface Methods For Process Optimization workshop. Call or visit our web site for information on content and schedules.

In this section, you will work with a three-factor central composite design on a chemical reaction. The factors are: time, temperature and catalyst. The experimenters measured two responses: yield and activity. You will optimize the process using models developed earlier in the Response Surface Tutorial.

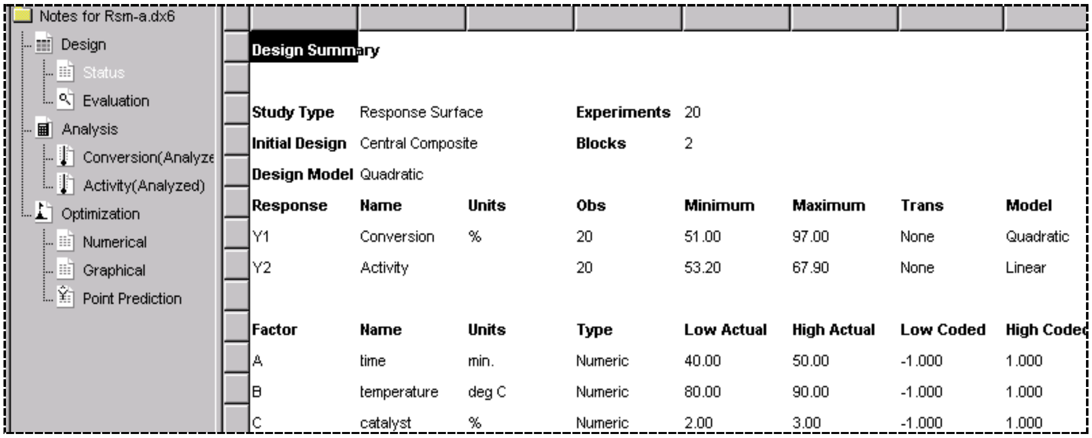
Start the program by finding and double clicking on the Design-Expert icon.

You will find the case study data, with the responses already analyzed, stored in a file named **Rsm-a.dx6**. Use the **File, Open Design** menu to load the data file. The standard file open dialog box appears.



File Open Dialog Box

Once you have found the proper drive, directory and file name, click on **Open** to load the data. To see a description of the design status, click on the **Status** node. Drag the left border and open the window to see the report better. You can also re-size columns with the mouse. From the design status screen you can see that we modeled conversion with a quadratic model and activity with a linear model.



Study Type	Response Surface		Experiments	20			
Initial Design	Central Composite		Blocks	2			
Design Model	Quadratic						
Response	Name	Units	Obs	Minimum	Maximum	Trans	Model
Y1	Conversion	%	20	51.00	97.00	None	Quadratic
Y2	Activity		20	53.20	67.90	None	Linear
Factor	Name	Units	Type	Low Actual	High Actual	Low Coded	High Coded
A	time	min.	Numeric	40.00	50.00	-1.000	1.000
B	temperature	deg C	Numeric	80.00	90.00	-1.000	1.000
C	catalyst	%	Numeric	2.00	3.00	-1.000	1.000

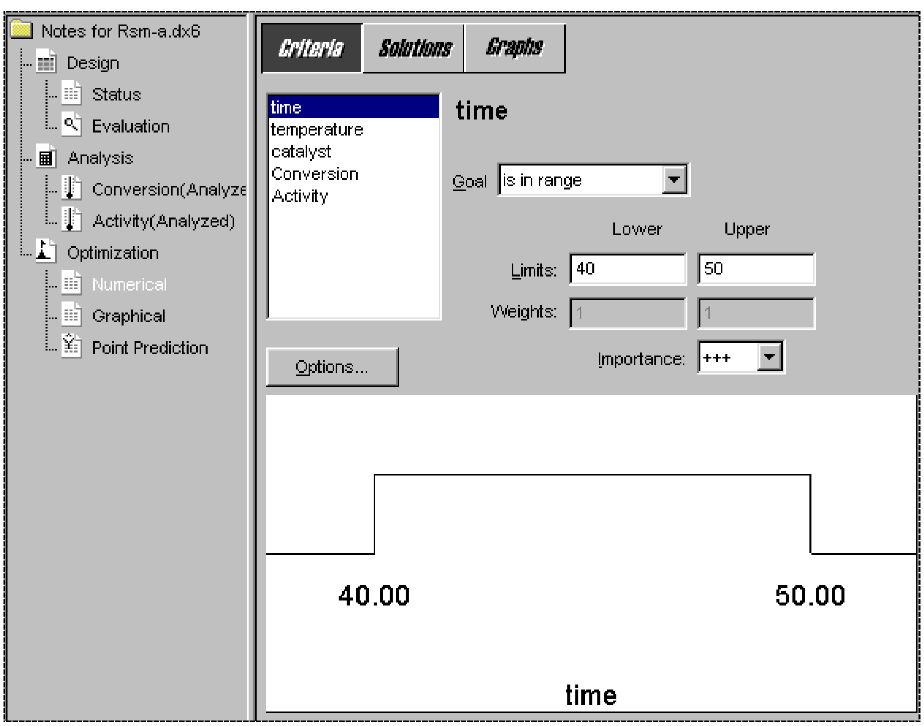
Design Status Screen

Numerical Optimization

Design-Expert software's numerical optimization will maximize, minimize or target:

- A single response
- A single response, subject to upper and/or lower boundaries on other responses
- Combinations of two or more responses.

We will lead you through the latter case: a multiple response optimization. Click on the **Numerical** node to start the process.



Setting Numeric Optimization Criteria

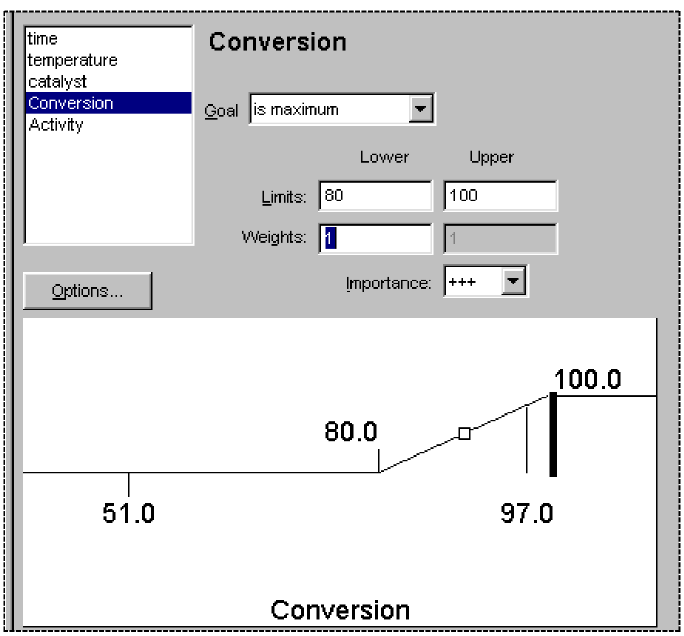
Design-Expert allows you to set criteria for all variables, including factors and propagation of error (POE). (We will get to POE later.) To be safe, the program sets the factor ranges to the actual levels (plus one to minus one in coded values). The limits for the responses default to the observed extremes. In this case, you should leave the settings for time, temperature and catalyst factors alone, but you will need to make some changes to the response criteria.

Now you get to the crucial phase of numerical optimization: assignment of "Optimization Parameters." The program uses five possibilities for a "Goal" to construct desirability indices (d_i):

- is maximum
- is minimum
- is target
- is in range
- is equal to (factors only)

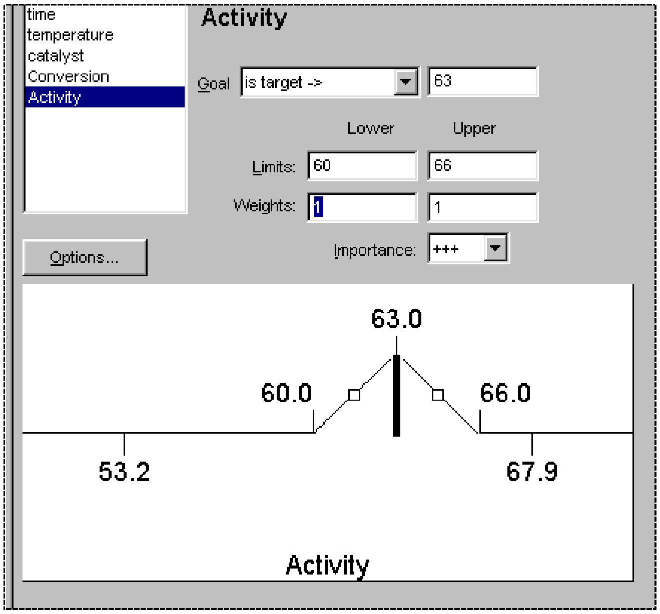
Desirabilities range from zero to one for any given response. The program combines the individual desirabilities into a single number and then searches for the greatest overall desirability. A value of one represents the ideal case. A zero indicates that one or more responses fall outside desirable limits. Design-Expert uses an optimization method developed by Derringer and Suich, described by Myers and Montgomery in *Response Surface Methodology*, John Wiley and Sons, New York (available from Stat-Ease).

For this tutorial case study, assume that you need the conversion to be as high as possible. Click on **Conversion** and set its **Goal** at **is maximum**. Set the **Lower Limit** to **80**, the lowest acceptable value. You must enter an **Upper Limit** to get the desirability equation to work properly, so set it at the theoretical high of **100**.



Conversion Criteria Settings

Now click on the second response, **Activity**. Set its **Goal** to **is target** of **63**. Enter a **Lower Limit** of **60** and an **Upper Limit** of **66**. These limits indicate that it is most desirable to achieve the targeted value of 63, but values in the range of 60-66 are acceptable. Values outside that range are not acceptable.



Activity Criteria Settings

These settings create the following desirability functions:

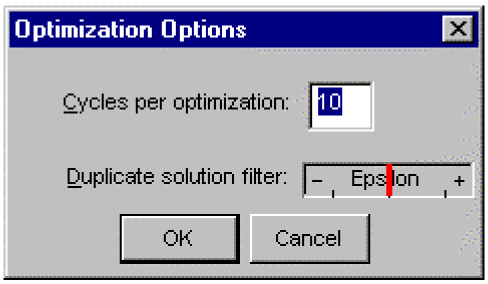
1. Conversion:
   - if less than 80%, then desirability (d_i) equals zero
   - from 80 to 100%, d_i ramps up from zero to one
   - if over 100%, then d_i equals one
2. Activity:
   - if less than 60, then d_i equals zero
   - from 60 to 63, d_i ramps up from zero to one
   - from 63 to 66, d_i ramps back down to zero
   - if greater than 66, then d_i equals zero

The user can select additional parameters, called weights, for each response. Weights give added emphasis to upper or lower bounds or emphasize a target value. With a weight of 1, the d_i will vary from 0 to 1 in linear fashion. Weights greater than 1 (maximum weight is 10) give more emphasis to the goal. Weights less than 1 (minimum weight is 0.1) give less emphasis to the goal. Leave the **Weights** fields at their default values of **1**.

Importance is a relative scale for weighting each of the resulting d_i in the overall desirability product. If you want to emphasize one response over the rest, set its importance higher than the other response importances. For this study, leave the **Importance** field at **+++**, a medium setting.

See the on-line help system for a more in-depth explanation of the construction of the desirability function, and formulas for the weights and importance.

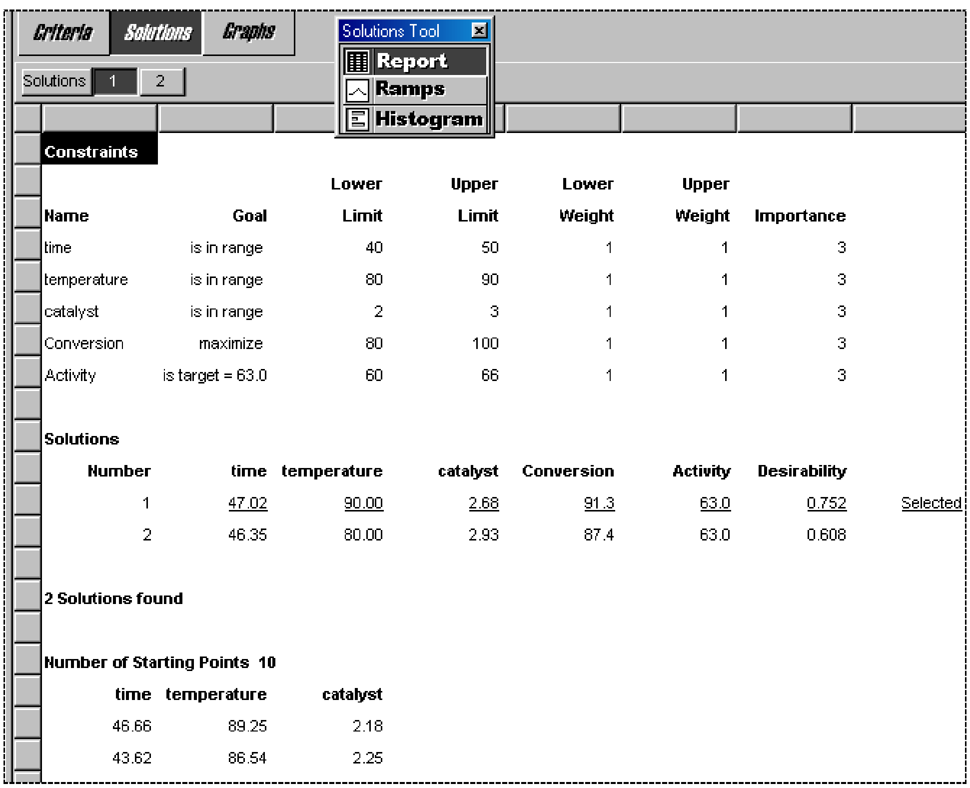
The **Options** button controls the number of cycles (searches) per optimization. If you have a very complex combination of response surfaces, increasing the number of cycles will give you more opportunities to find the optimal solution. The Duplicate solution filter establishes the epsilon (minimum difference) for eliminating duplicate solutions. Leave these options at their default levels (shown below).



Optimization Options Dialog Box

Running the optimization

Start the optimization by clicking on the ***Solutions*** icon.



Constraints

Name	Goal	Lower Limit	Upper Limit	Lower Weight	Upper Weight	Importance
time	is in range	40	50	1	1	3
temperature	is in range	80	90	1	1	3
catalyst	is in range	2	3	1	1	3
Conversion	maximize	80	100	1	1	3
Activity	is target = 63.0	60	66	1	1	3

Solutions

Number	time	temperature	catalyst	Conversion	Activity	Desirability	
1	47.02	90.00	2.68	91.3	63.0	0.752	Selected
2	46.35	80.00	2.93	87.4	63.0	0.608	

2 Solutions found

Number of Starting Points 10

time	temperature	catalyst
46.66	89.25	2.18
43.62	86.54	2.25

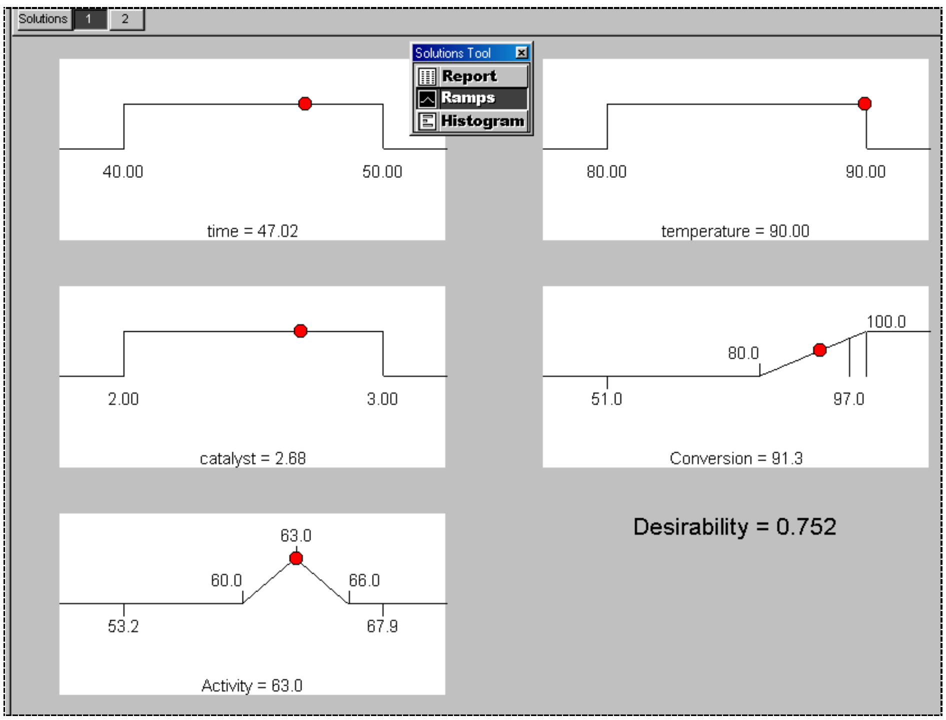
Numerical Optimization Report on Solutions (Your results may differ)

The program randomly picks a set of conditions from which to start its search for desirable results. Multiple cycles improve the odds of finding multiple local optimums, some of which will be higher in desirability than others. After grinding through 10 cycles of optimization, the results appear.

The report view shows the results in tabular form. Due to the random starting conditions, your results are likely to be slightly different from those shown here. Note that the last solution falls short of the first for conversion. There may be some duplicates in between. These passed through the filter discussed earlier. If you want to adjust the filter, go to the Options button and change the Duplicate Solutions Filter. If you move the Filter bar to the right you will decrease the number of solutions shown. Likewise, moving the bar to the left increases the number of solutions.

The Solutions tool provides three views of the same optimization. (Drag the tool to a convenient location on the screen.)

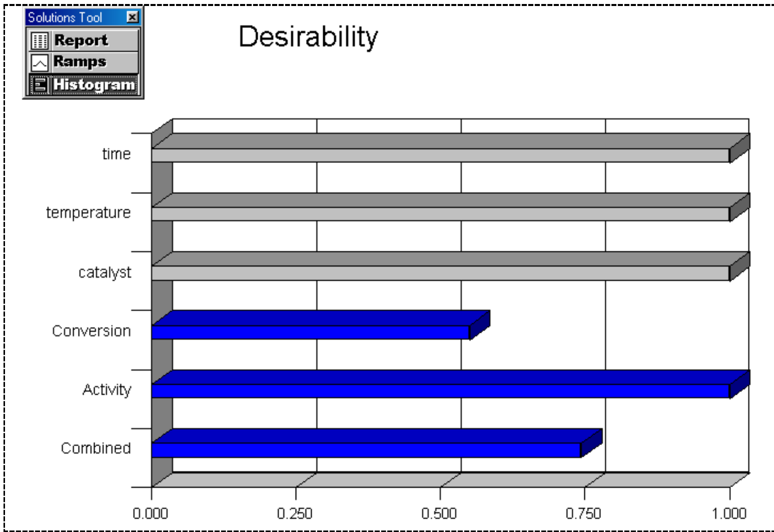
Click on the solutions view option **Ramps**.



Ramps Report on Numerical Optimization

The ramp display combines the individual graphs for easier interpretation. The dot on each ramp reflects the factor setting or response prediction for that solution. The height of the dot shows how desirable it is. Press the different solution buttons (1, 2, 3,…) and watch the dots. They may move only very slightly from one solution to the next. However, if you look closely at temperature, you should find two distinct optimums, one near 80 degrees and the other near 90 degrees. You may see slight differences in the results due to variations in approach from the various random starting points.

Select the **Histogram** view.

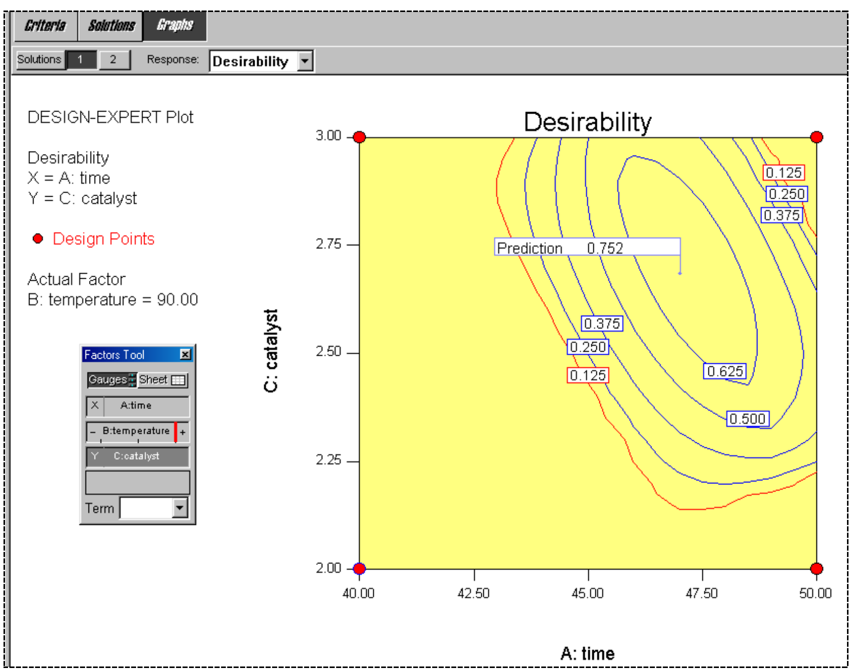


Solution to Multiple Response Optimization - Desirability Histogram

The histogram shows how well each variable satisfied the criteria: values near one are good. Nearly duplicate solutions, as found by the duplicate solutions filter, will be eliminated. In this example, you will find two or more somewhat different solutions. You can cycle through each of the solutions: the best is listed first.

Optimization Graphs

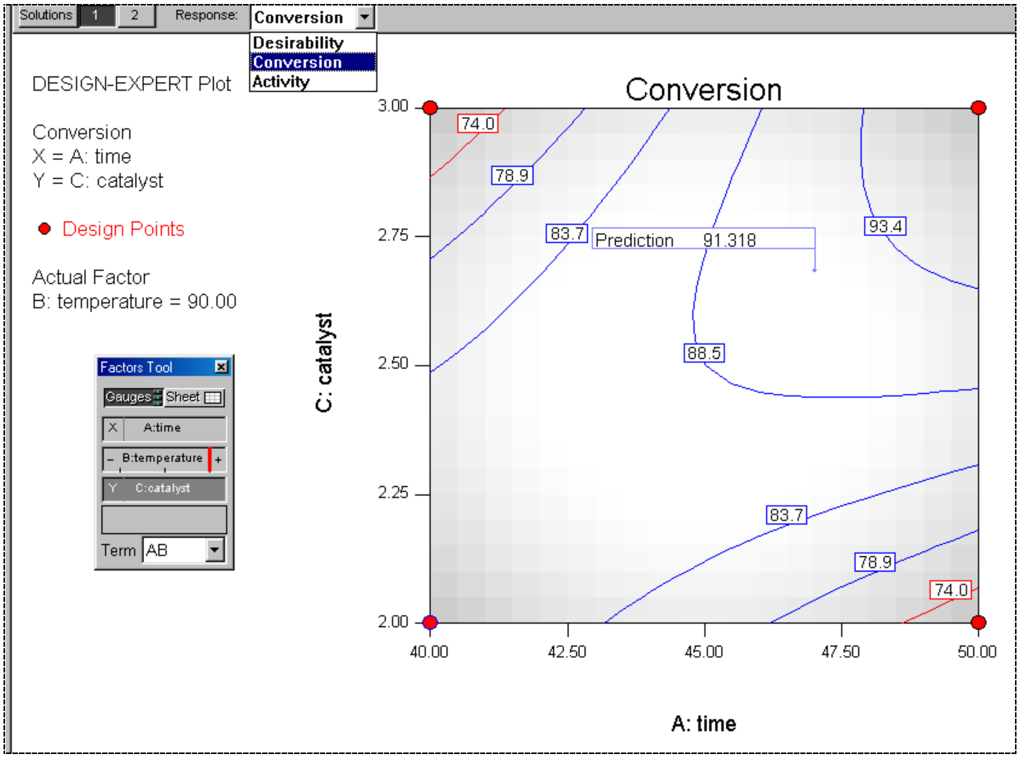
Press ***Graphs*** to view the results of the first optimization run. Go to the **Factor** control and right click on **catalyst**. Make it the **Y axis**. Temperature then becomes a constant factor at 90 degrees.



Desirability Graph (After Changing Axes)

Design-Expert software sets a flag at the optimal point. You can plant additional flags by doing a right mouse click at any location. Right click again on the flag and **Toggle size** to see the associated desirability value and the factor levels.

To view the responses associated with the desirability, select the desired **Response** from the drop down list. Take a look at the plot for **Conversion**.

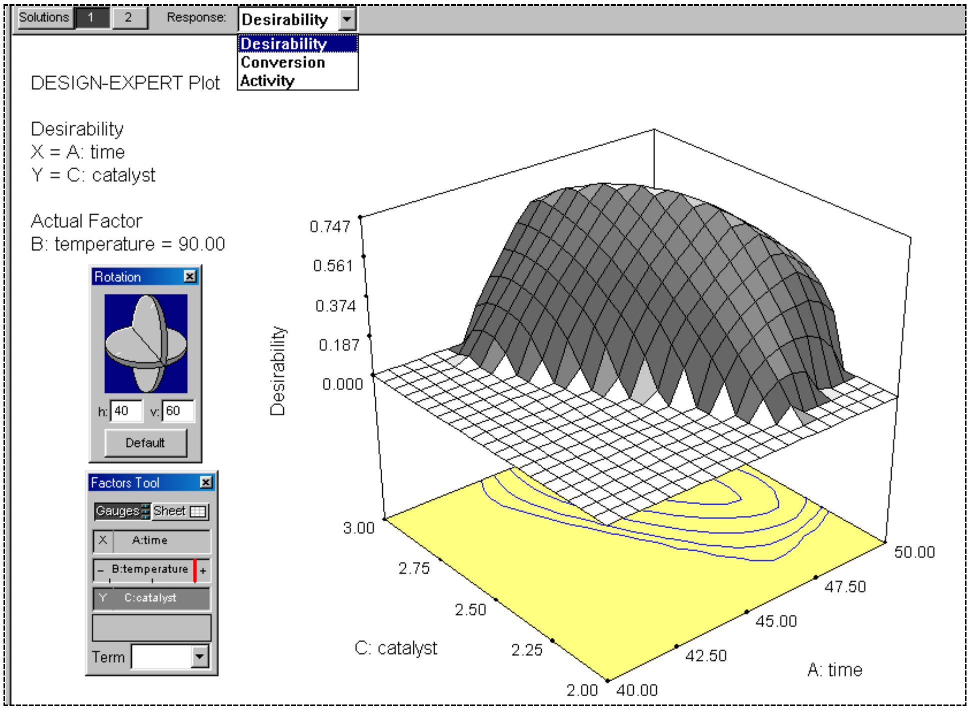


Conversion Contour Plot (with optimum flagged)

If you like, look at the optimal activity response as well.

To look at the desirability surface in three dimensions, click again on **Response** and choose **Desirability**. Then select **View, 3D Surface** from the main menu. Drag the tools out of the way as needed to view the results. Then rotate the plot for a different perspective by using the **Rotation** tool. Drag the rim of the wheels with your mouse pointer to change the orientation of the 3D plot.

You can change the quality of the 3D graph by going to the **Edit, Preferences** menu item, or by doing a right mouse click on the graph. Click on the **Graph** tab and then set parameters as you desire. Give this a try. Then if you have a printer attached, make a hard copy by doing a **File**, **Print**. Show your colleagues what Design-Expert software will do!

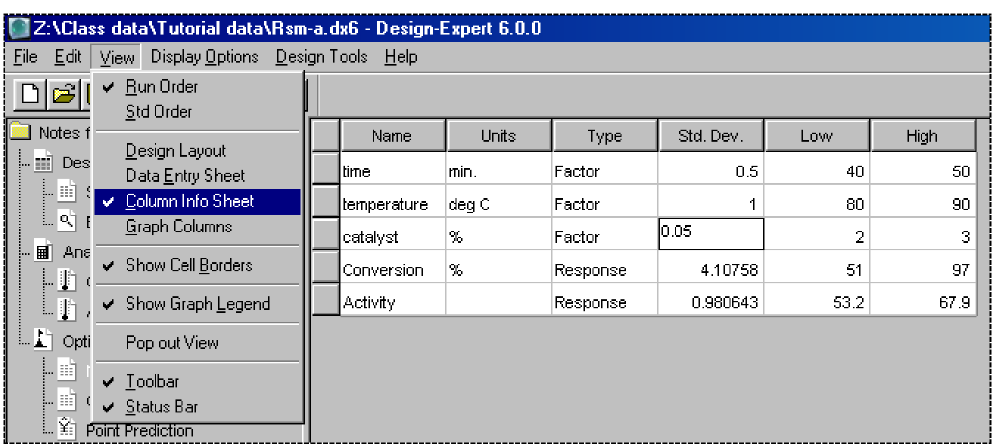


3D Desirability Plot

Adding Propagation of Error (POE) to the Optimization

If you have prior knowledge of the variation in your input factors, this information can be fed into Design-Expert software. Then you can generate propagation of error (POE) plots that show how that error is transmitted to the response. Look for conditions that minimize the transmitted variation, thus creating a process that's robust to the factor settings.

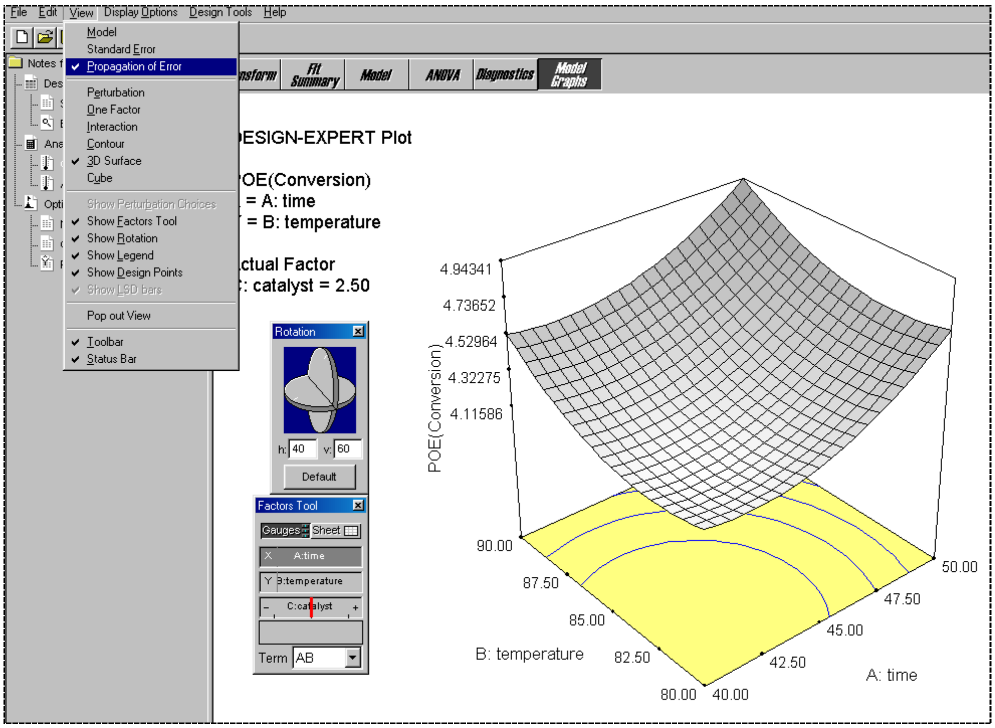
Start by clicking on the **Design** node on the left side of the screen to get back to the design layout. Then select **View, Column Info Sheet.** Enter the following information into the Std. Dev. column: time: **0.5**, temperature: **1.0**, catalyst: **0.05**, as shown on the screen below.



Column Info Sheet with Factor Standard Deviations Filled In

Notice that the software already entered the standard deviation for the analyzed response, Conversion (4.10). Since you haven't changed any other data, the software will remember your previous analysis choices and you can simply click through the analysis buttons. This time the Propagation of Error (POE) graph, which was grayed out before, will be available from the Model Graph node.

Click on the **Conversion** analysis node on the left to start the analysis again. Then jump past the intermediate buttons for analysis and click on the ***Model Graphs*** button. Select **View, Propagation of Error** and **View, 3D Surface**.

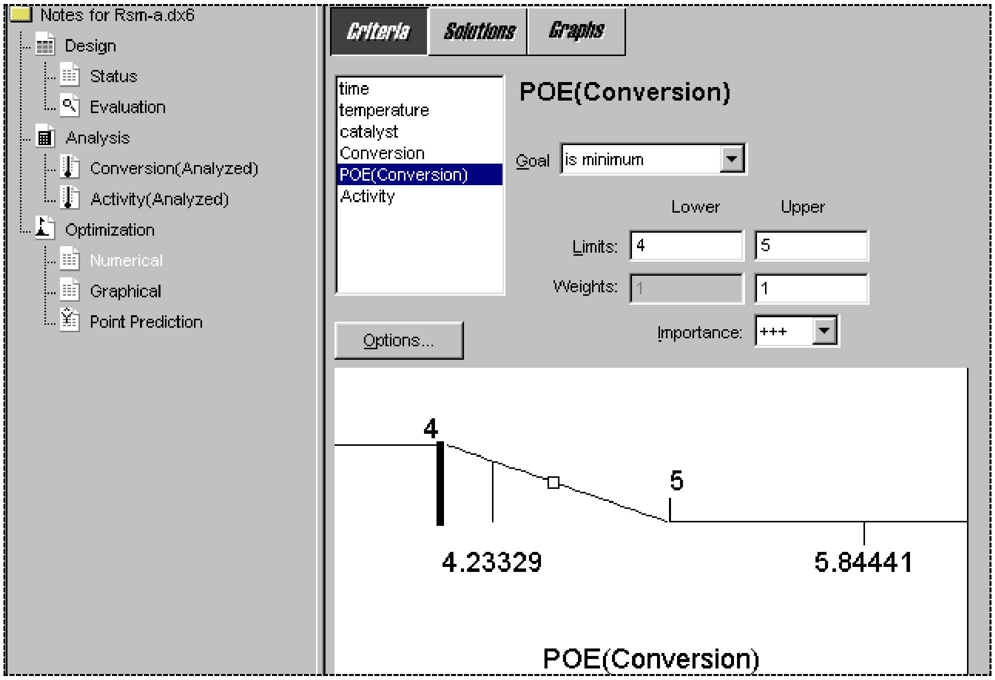


3D Surface view of the POE Graph

The lower the POE the better, because less of the error in control factors will be transmitted to the selected response. The end result is a more robust process. However, POE will only work when the response surface is non-linear, such as for the Conversion. When the surface is linear, such as that for Activity, the error will be transmitted equally throughout the region.

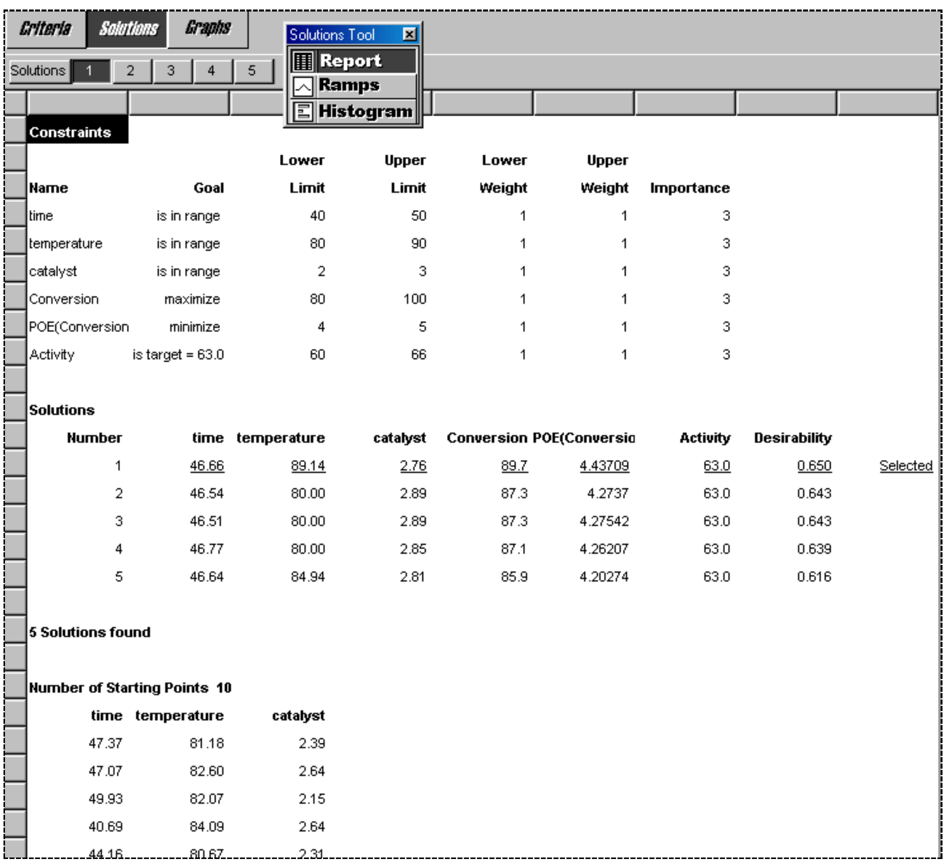
For an in-depth explanation of POE, attend Stat-Ease's workshop Robust Design, DOE Tools for Reducing Variability. Call, or check our web site for a schedule.

Now that you've found optimum conditions for the two responses, let's go back and add criteria for the propagation of error. Click on **Numerical** optimization node. Set the **POE (Conversion) Goal** to **is minimum** with a **Lower Limit** of **4** and an **Upper Limit** of **5** as shown below.



Set Goal and Limits for POE (Conversion)

Now click on the ***Solutions*** node to generate new solutions with the additional criteria.



Constraints

Name	Goal	Lower Limit	Upper Limit	Lower Weight	Upper Weight	Importance
time	is in range	40	50	1	1	3
temperature	is in range	80	90	1	1	3
catalyst	is in range	2	3	1	1	3
Conversion	maximize	80	100	1	1	3
POE(Conversion	minimize	4	5	1	1	3
Activity	is target = 63.0	60	66	1	1	3

Solutions

Number	time	temperature	catalyst	Conversion	POE(Conversio	Activity	Desirability	
1	46.66	89.14	2.76	89.7	4.43709	63.0	0.650	Selected
2	46.54	80.00	2.89	87.3	4.2737	63.0	0.643	
3	46.51	80.00	2.89	87.3	4.27542	63.0	0.643	
4	46.77	80.00	2.85	87.1	4.26207	63.0	0.639	
5	46.64	84.94	2.81	85.9	4.20274	63.0	0.616	

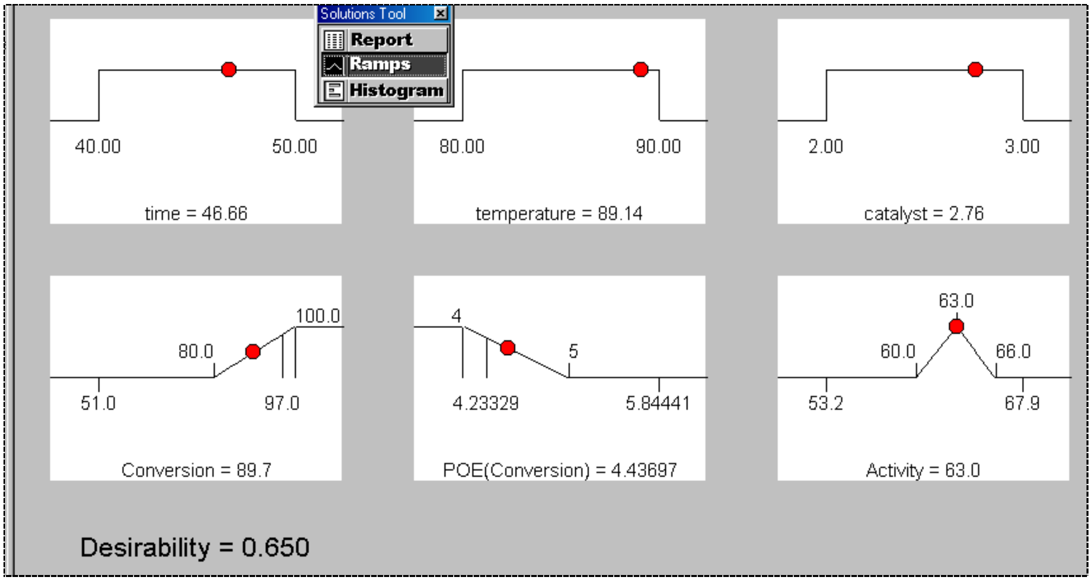
5 Solutions found

Number of Starting Points 10

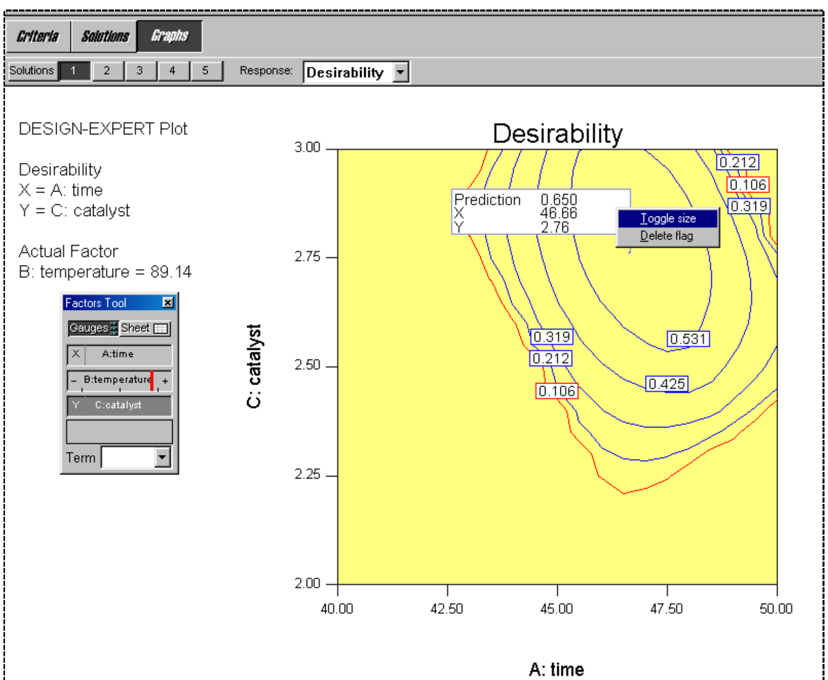
time	temperature	catalyst
47.37	81.18	2.39
47.07	82.60	2.64
49.93	82.07	2.15
40.69	84.09	2.64
44.16	80.67	2.31

Solutions Generated with Added POE Criteria (Your results may differ)

Click on the solutions view option **Ramps**. Click on the alternate solutions (2, 3,…). Watch the red dots. What changes?



Select the ***Graphs*** node. Click the number **1** solution. If you haven't already, choose **View, Contour**. To make the view similar to what we had before, on the **Factors** tool palette change **catalyst** to the **Y axis**. Also right-click on the flag and **Toggle Size**. This solution represents the formulation that best maximizes conversion and achieves a target value of 63 for activity, while at the same time finds the spot with the minimum error transmitted to the responses. So, this should represent process conditions that are robust to slight variations in the factor settings.



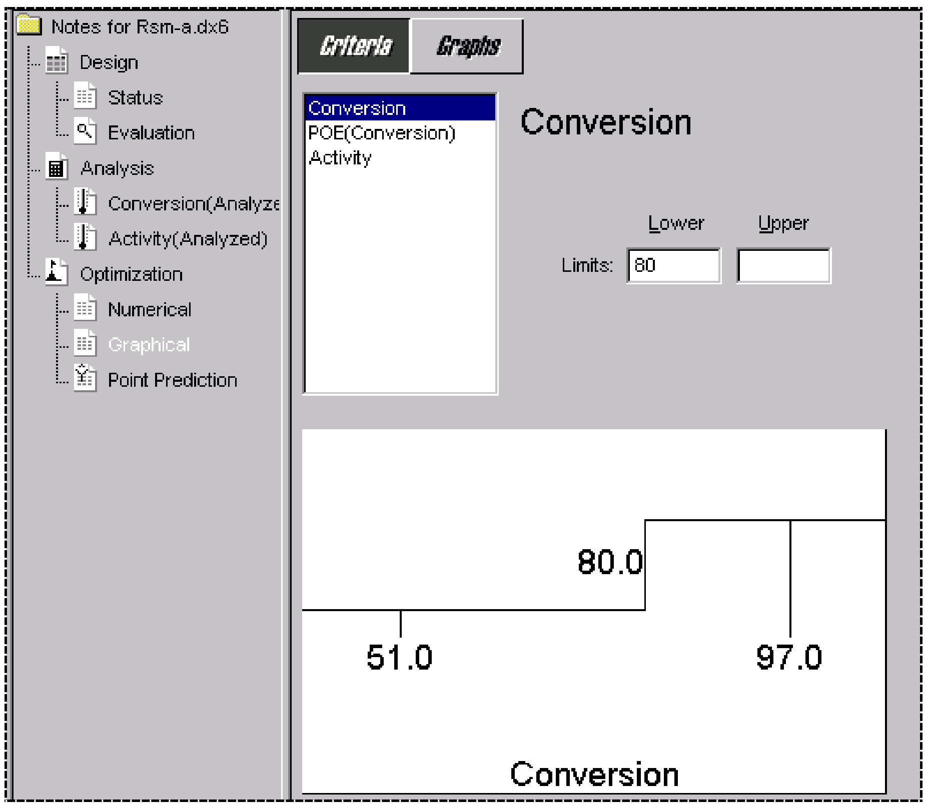
Optimal Solution with Added POE Criteria

Graphical Optimization

When you did the numerical optimization, you found an area of satisfactory solutions at a temperature of 90 degrees. To see a broader operating window, click on the **Graphical** node. The requirements are essentially the same as in the numerical optimization:

- 80 < Conversion
- POE(Conversion) < 5
- 60 < Activity < 66

For **Conversion**, which comes up by default, enter **80** for the **Lower Limit**. You do not need to enter a high limit for the graphical optimization to work.

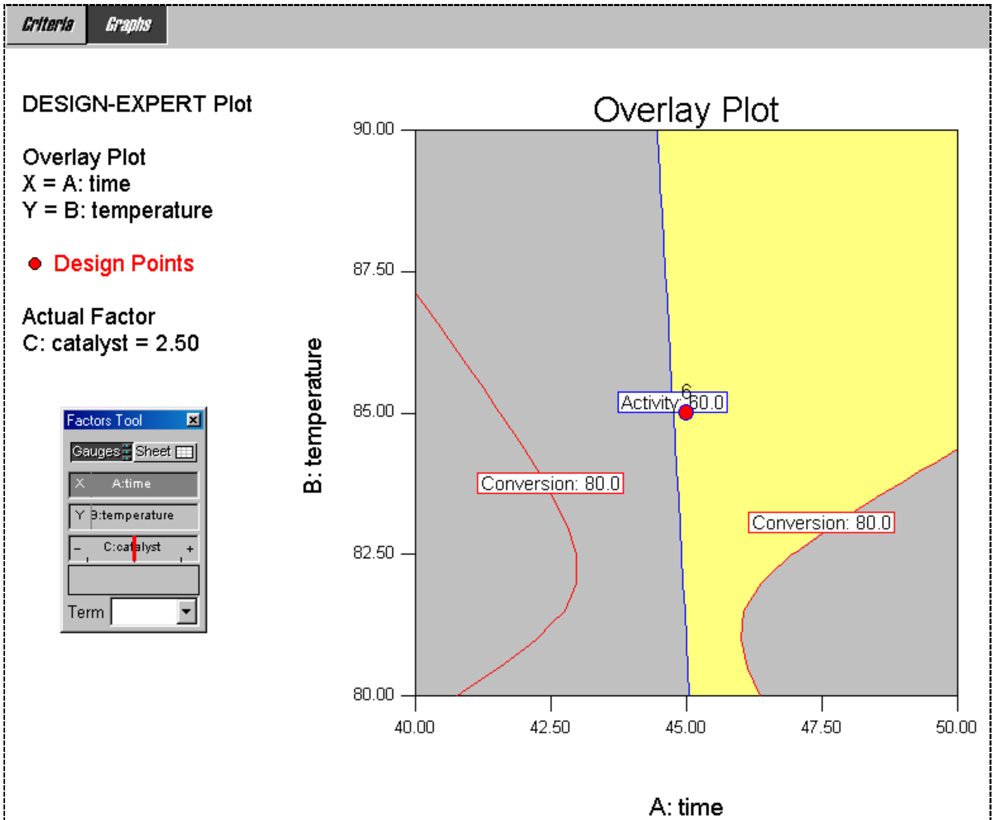


Completed Screen for Graphical Optimization: Conversion

Click on the response for **POE(Conversion)**. Enter **5** for the **Upper Limit**.

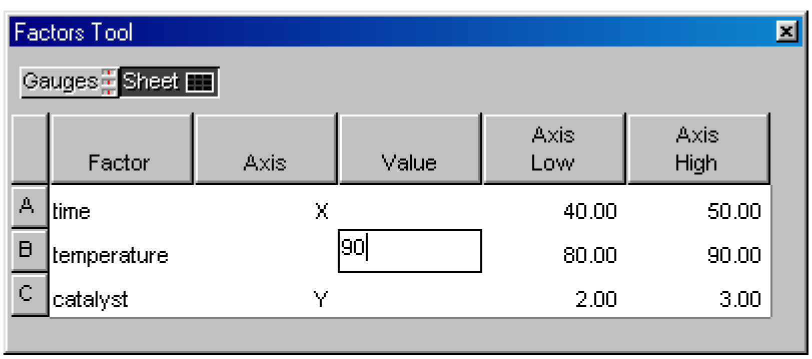
Finally, click on the response of **Activity**. Enter **60** for the **Lower Limit** and **66** for the **Upper Limit**.

Then click on the ***Graphs*** button to produce the “overlay” plot. Notice that regions not meeting your specifications are shaded out, leaving (hopefully!) an operating window or “sweet spot.”



Overlay Plot

To match the numerical optimization, go to the **Factors** tool, right click on **catalyst** and select it for the **Y-axis**. Then press the **Sheet** option and set **temperature** as a constant **Value** at **90**.



	Factor	Axis	Value	Axis Low	Axis High
A	time	X		40.00	50.00
B	temperature		90	80.00	90.00
C	catalyst	Y		2.00	3.00

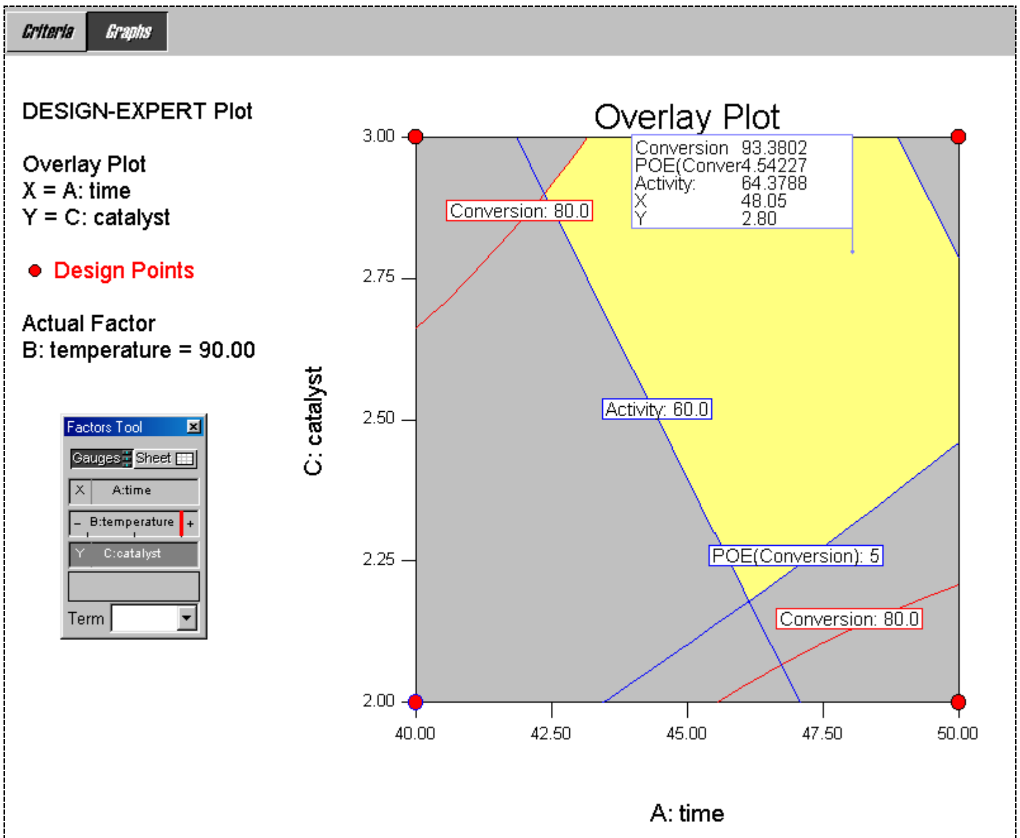
Sheet Mode to Set Constant for Temperature

Design-Expert now overlays response contours for each response. Click on the **Gauges** option to make the **Factors Tool** less obtrusive. Now you can search for a "compromise" optimum - windows of operability where requirements simultaneously meet the critical properties.

You can optimize a virtually unlimited number of responses on the same graph. However, they must first be analyzed. In the first tutorial you developed a quadratic model for conversion and a linear model for activity. Only those responses for which you enter lower and/or upper levels will be plotted.

The shaded areas on the graphical optimization plot do <u>not</u> meet the selection criteria. The lines mark the high or low boundaries on the responses. The clear (or colored yellow if you haven't made changes to your color scheme) "window" shows where you can set the factors to satisfy the requirements on both responses.

Move the mouse pointer into the clear (or yellow) area. Right click to add a flag. The flag shows the predicted value of conversion, the propagation of error for conversion and activity, at that point on the plot.



Graphical Optimization Plot, Axes Changed, Flag Added

Plant multiple flags if you like. Delete flags by right clicking on them and selecting **Delete flag**. You can also change criteria "on the fly" by clicking and dragging the contour lines. You can explore this window for high values of conversion. You will find the maximum conversion in the 90-degree slice is about 96%.

Graphical optimization works great for two factors, but as the factors increase, it becomes more and more tedious. You will find solutions much more quickly by using the numerical optimization feature. Then return to the graphical optimization and produce outputs for presentation purposes.

Final Comments

We feel that numerical optimization provides powerful insights when combined with graphical analysis. Numerical optimization becomes essential when you investigate many components with many responses. However, computerized optimization will not work very well in the absence of subject matter knowledge. For example, a naive user may define impossible optimization criteria. The result will be zero desirability everywhere! To avoid this, try setting broad acceptable ranges. Narrow them down as you gain knowledge about how changing factor levels affect the responses. Often, you will need to make more than one pass to find the “best” factor levels to satisfy constraints on several responses simultaneously.

The Response Surface Optimization Tutorial completes the basic introduction to doing RSM with Design-Expert software. If you want to learn more about the methods shown in this tutorial, attend the Stat-Ease workshop Response Surface Methods For Process Optimization.

We appreciate your questions and comments on Design-Expert software. Call us, send an annotated fax of output, write us a letter or send us an e-mail. You will find our address, phone number and web site listed in the last part of the Introduction to this manual.