Chapter 13. Supplemental Text Material

13-1. The Staggered, Nested Design

In Section 13-1.4 we introduced the staggered, nested design as a useful way to prevent the number of degrees of freedom from "building up" so rapidly at lower levels of the design. In general, these designs are just unbalanced nested designs, and many computer software packages that have the capability to analyze general unbalanced designs can successfully analyze the staggered, nested design. The general linear model routine in Minitab is one of these packages.

To illustrate a staggered, nested design, suppose that a pharmaceutical manufacturer is interested in testing the absorption of a drug two hours after the tablet is ingested. The product is manufactured in lots, and specific interest focuses on determining whether there is any significant lot-to-lot variability. Excessive lot-to-lot variability probably indicates problems with the manufacturing process, perhaps at the stage where the coating material that controls tablet absorption is applied. It could also indicate a problem with either the coating formulation, or with other formulation aspects of the tablet itself.

The experimenters select a = 10 lots at random from the production process, and decide to use a staggered, nested design to sample from the lots. Two samples are taken at random from each lot. The first sample contains two tablets, and the second sample contains only one tablet. Each tablet is test for the percentage of active drug absorbed after two hours. The data from this experiment is shown in Table 1 below.

Table 1. The Drug Absorption Experiment

	Sample	
Lot	1	2
1	24.5, 25.9	23.9
2	23.6, 26.1	25.2
3	27.3, 28.1	27.0
4	28.3, 27.5	27.4
5	24.3, 24.1	25.1
6	25.3, 26.0	24.7
7	27.3, 26.8	28.0
8	23.3, 23.9	23.0
9	24.6, 25.1	24.9
10	24.3, 24.9	25.3

The following output is from the Minitab general linear model analysis procedure.

General Linear Model

```
Factor        Type Levels Values
Lot          random     10  1  2  3  4  5  6  7  8  9 10
Sample(Lot)  random     20 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2

Analysis of Variance for Absorp., using Adjusted SS for Tests

Source         DF     Seq SS     Adj SS     Adj MS       F      P
Lot             9    58.3203    52.3593     5.8177   14.50  0.000
Sample(Lot)    10     4.0133     4.0133     0.4013    0.71  0.698
Error          10     5.6200     5.6200     0.5620
Total          29    67.9537

Expected Mean Squares, using Adjusted SS

Source          Expected Mean Square for Each Term
 1 Lot          (3) +  1.3333(2) +  2.6667(1)
 2 Sample(Lot)  (3) +  1.3333(2)
 3 Error        (3)

Error Terms for Tests, using Adjusted SS

Source          Error DF  Error MS  Synthesis of Error MS
 1 Lot             10.00    0.4013  (2)
 2 Sample(Lot)     10.00    0.5620  (3)

Variance Components, using Adjusted SS

Source       Estimated Value
Lot                   2.0311
Sample(Lot)          -0.1205
Error                 0.5620
```

As noted in the textbook, this design results in a - 1 = 9 degrees of freedom for lots, and a = 10 degrees of freedom for samples within lots and error. The ANOVA indicates that there is a significant difference between lots, and the estimate of the variance component for lots is $\hat{\sigma}^2_{Lots} = 2.03$. The ANOVA indicates that the sample within lots is not a significant source of variability. This is an indication of lot homogeneity. There is a small negative estimate of the sample-within-lots variance component. The experimental error variance is estimated as $\hat{\sigma}^2 = 0.526$. Notice that the constants in the expected mean squares are not integers; this is a consequence of the unbalanced nature of the design.

13-2. Inadvertent Split-Plots

In recent years experimenters from many different industrial settings have become exposed to the concepts of designed experiments, either from university-level DOX courses or from industrial short courses and seminars. As a result, factorial and fractional factorial designs have enjoyed expanded use. Sometimes the principle of randomization is not sufficiently stressed in these courses, and as a result experimenters may fail to understand its importance. This can lead to **inadvertent split-plotting** of a factorial design.

For example, suppose that an experimenter wishes to conduct a 2^4 factorial using the factors A = temperature, B = feed rate, C = concentration, and D = reaction time. A 2^4 with the runs arranged in random order is shown in Table 1.

Table 1. A 2^4 Design in Random Order

Std	Run	Block	Factor A: Temperature DegC	Factor B: Feed rate gal/h	Factor C: Concentration gm/l	Factor D: Reaction time h	Response Yield
16	1	Block 1	150	8	30	1.2	
9	2	Block 1	100	5	25	1.2	
7	3	Block 1	100	8	30	1	
12	4	Block 1	150	8	25	1.2	
2	5	Block 1	150	5	25	1	
13	6	Block 1	100	5	30	1.2	
1	7	Block 1	100	5	25	1	
10	8	Block 1	150	5	25	1.2	
3	9	Block 1	100	8	25	1	
14	10	Block 1	150	5	30	1.2	
6	11	Block 1	150	5	30	1	
4	12	Block 1	150	8	25	1	
5	13	Block 1	100	5	30	1	
15	14	Block 1	100	8	30	1.2	
11	15	Block 1	100	8	25	1.2	
8	16	Block 1	150	8	30	1	

When the experimenter examines this run order, he notices that the level of temperature is going to start at 150 degrees and then be changed eight times over the course of the 16 trials. Now temperature is a hard-to-change-variable, and following every adjustment to temperature several hours are needed for the process to reach the new temperature level and for the process to stabilize at the new operating conditions.

The experimenter may feel that this is an intolerable situation. Consequently, he may decide that fewer changes in temperature are required, and rearange the temperature levels in the experiment so that the new design appears as in Table 2. Notice that only three changes in the level of temperature are required in tis new design. In efect, the experimenter will set the temperature at 150 degrees and perform four runs with the other three factors tested in random order. Then he will change the temperature to 100 degrees and repeat the process, and so on. The experimenter has inadvertently introduced a split-plot structure into the experiment.

Table 2. The Modified 2^4 Factorial

Std	Run	Block	Factor A: Temperature DegC	Factor B: Feed rate gal/h	Factor C: Concentration gm/l	Factor D: Reaction time h	Response Yield
16	1	Block 1	150	8	30	1.2	
9	2	Block 1	150	5	25	1.2	
7	3	Block 1	150	8	30	1	
12	4	Block 1	150	8	25	1.2	
2	5	Block 1	100	5	25	1	
13	6	Block 1	100	5	30	1.2	
1	7	Block 1	100	5	25	1	
10	8	Block 1	100	5	25	1.2	
3	9	Block 1	150	8	25	1	
14	10	Block 1	150	5	30	1.2	
6	11	Block 1	150	5	30	1	
4	12	Block 1	150	8	25	1	
5	13	Block 1	100	5	30	1	
15	14	Block 1	100	8	30	1.2	
11	15	Block 1	100	8	25	1.2	
8	16	Block 1	100	8	30	1	

Typically, most inadvertent split-plotting is not taken into account in the analysis. That is, the experimenter analyzes the data as if the experiment had been conducted in random order. Therefore, it is logical to ask about the impact of ignoring the inadvertent split-plotting. While this question has not been studied in detail, generally inadvertently running a split-plot and not properly accounting for it in the analysis probably does not have major impact **so long as the whole plot factor effects are large**. These factor effect estimates will probably have larger variances that the factor effects in the subplots, so part of the risk is that small differences in the whole-plot factors may not be detected. Obviously, the more systematic fashion in which the whole-plot factor temperature was varied in Table 2 also exposes the experimenter to confounding of temperature with some nuisance variable that is also changing with time. The most extreme case of this would occur if the first eight runs in the experiment were made with temperature at the low level (say), followed by the last eight runs with temperature at the high level.

13-3. Fractional Factorial Experiments in Split-Plots

Occasionally a fractional factorial experiment will be conducted in a split-plot. As an illustration consider the experiment described in Problem 8-7. Pignatiello and Ramberg (1985) discussed this experiment. It is a 2^{5-1} fractional factorial performed to study the effect of heat treating process variables on the height of truck springs. The factors are *A*

= transfer time, *B* = heating time, *C* = oil quench temperature*, D* = temperature, and *E* = hold down time. Suppose that factors *A*, *B*, and *C* are very hard to change and the other two factors, *D* and *E* are easy to change. For example, it might be necessary to first manufacture the springs by varying factors *A* – *C*, then hold those factors fixed while varying factors *D* and *E* in a subsequent experiment.

We consider a modification of that experiment, because the original experimenters may not have run it as a split-plot, and because they did not use the design generator that we are going to use. Let the whole-plot factors be denoted by *A*, *B*, and *C* and the split-plot factors be denoted by *D** and *E**. We will select the design generator *E* = ABCD**. The layout of this design has eight whole plots (the eight combinations of factors *A*, *B*, and *C* each at two levels). Each whole plot is divided into two subplots, and a combination of the treatments *D* and *E* are tested in each split plot (the exact treatment combinations depend on the signs on the treatment combinations for the whole plot factors through the generator).

Assume that all three, four, and five factor interactions are negligible. If this assumption is reasonable, then in the whole plot all whole plot factors *A*, *B*, and *C* and their two-factor interactions can be estimated. They would be tested against the whole plot error. Alternatively, if the design is unreplicated, their effects could be assessed via a normal probability plot or possibly by Lenth's method. The subplot factors *D** and *E** and their two-factor interaction *D*E** can also be estimated. There are six two-factor interactions of whole-plot and split-plot factors that can also be estimated, *AD**, *AE**, *BD**, *BE**, *CD**, and *CE**. In general, it turns out that any split-plot main effect or interaction that is aliased with whole-plot main effects or interactions involving only whole-plot factors would be compared to the whole plot error. Further, split-plot main effects or interactions involving at least one split-plot factor that are not aliased with whole-plot main effects or interactions involving only whole-plot factors are be compared to the split-plot error. See Bisgaard (1992) for a thorough discussion. Therefore, in our problem, all of the effects *D**, *E*, *D*E**, *AD**, *AE**, *BD**, *BE**, *CD**, and *CE** are all compared to the split-plot error. Alternatively, they could be assessed via a normal probability plot or possibly by Lenth's method.

Recently, several papers have appeared on the subject of fractional factorials in split-plots. Bisgaard (2000) is highly recommended. See also Bingham and Sitter (1999) and Huang, Chen and Voelkel (1999).

Supplemental References

Bingham, D. and R. R. Sitter (1999). "Minimum Aberration Two-Level Fractional Factorial Split-Plot Designs". *Technometrics*, Vol. 41, pp. 62-70.

Bisgard, S. (2000). "The Design and Analysis of $2^{k-p} \times 2^{q-r}$ Split Plot Experiments". *Journal of Quality Technology*, Vol. 32, pp. 39-56.

Huang, P., D. Chen and J. Voelkel (1999). "Minimum Aberration Two-Level Split-Plot Designs". *Technometrics*, Vol. 41, pp. 314-326.